Air Traffic Flow Management at Airports: A Unified Optimization Approach

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Motivation for *Unified Airport Optimization*

- Airports = key bottlenecks
- Current approaches in general solve sub-problems in isolation
  - Sub-optimality
  - Overall infeasibility

⇒ *Unified Approach*
Key Sub-problems at Airports
Runway Sequencing

- Minimum separation between flights
- Depends on flight types
  - Arrival, departure; heavy, B-757, large, small
Runway Sequencing – Literature

• TSP/TRP
• Single runway
• Constrained position shifting
• Dynamic programming
• Time windows, side constraints
• Stochastic runway scheduling

• Dear (1976)
• Psaraftis (1980)
• Trivizas (1987)
• Balakrishnan and Chandran (2010)
• Clare and Richards (2011)
• Sölveling et al. (2011)
Runway Configuration Management

- Rules regulating runway use
- *Runway configuration* =
  - Combination of runways
  - Operating modes
- Which configuration?
- When?
- Literature:
  - Bertsimas, Frankovich, and Odoni (2011)
Flight Routing

• Route flights to assigned runway and beyond
• Determine gate-holding/speed control, if any

*Diagram thanks to Dr Tom Reynolds
Flight Routing – Literature

- **Surface management:**
  - Feron et al. (1997), Pujet et al. (1999), Carr (2001), Burgain (2010), Simaiakis et al. (2011)

- **Taxi route optimization:**
  - Marín (2006), Rathinam et al. (2008), Roling and Visser (2008), Malik et al. (2010)

- **Sequencing & taxiing:**
  - Gotteland et al. (2009), Clare and Richards (2011)
Airport Operations Optimization Problem (AOOP)

- Selecting a runway configuration sequence
- Determining the service rate of arrivals and departures
- Assigning flight types to runways and determining their sequence
- Determining the gate-holding duration of departures
- Routing flights to their assigned runway and onwards within the terminal area
Summary of Approach

Two phases, each a binary optimization problem

• Decomposition motivation:
  – key bottleneck of system at runways
  – so initially ignore capacity of gates, taxiways, airspace

• Phase I (Configurations, sequencing, assignment)
  – Optimal under assumption

• Phase II (Routing)
  – Uses phase one solution
  – Tractably solves AOOP without assumption
Phase I Decisions

Configuration Selection

“Northeast Flow”

Runway Assignment & Sequencing

4L

9

4R
Phase I – Key Data

• Time horizon 1,...,T, @20s intervals
• Configuration availability $U_t$
• For each flight f:
  – A flight type i
  – Set of feasible runways $R_f$
  – For each runway r in $R_f$:
    • Earliest possible takeoff/landing time $T^f_r$
    • Computed based on shortest paths
• Objective – weighted cost of delays
Phase I – Decision Variables

\[ \omega_{kt} = 1 \iff \text{configuration k is used at time t} \]
\[ \chi_t = 1 \iff \text{we change configuration at time t} \]
\[ \varphi_r^f = 1 \iff \text{flight f is assigned to runway r} \]
\[ \psi_{rt}^i = 1 \iff \text{a flight of type i reaches runway r at t} \]

• Naïve approach: \( \psi_{rt}^f \)
  – Separation constraints
  – Model size

• Flight-slot assignment guaranteed
\[
\begin{align*}
\min & \quad \Psi \\
\text{s.t.} & \quad \sum_{k \in K} \omega_{kt} = 1, \quad \forall t \in \mathcal{T}, \\
& \quad \psi_{rt}^i = 0, \quad \forall i \in \mathcal{C}, \quad r \in \mathcal{U}_i, \quad t \in \mathcal{T}, \\
& \quad \psi_{r,t}^i + \psi_{r,t-h}^j \leq 1, \quad \forall i, j \in \mathcal{C}, \quad r \in \mathcal{R}_i \cap \mathcal{R}_j, \quad h \in \{1, \ldots, \min\{s_{ij}^r - 1, t - 1\}\}, \quad t \in \mathcal{T} \setminus \{1\}, \\
& \quad \psi_{r,t-h}^i + \psi_{r',t}^j \leq 1, \quad \forall i, j \in \mathcal{C}, \quad (r, r') \in (\mathcal{R}_i \times \mathcal{R}_j) \cap \mathcal{V}, \quad h \in \{0, \ldots, \min\{s_{ij}^{(r,r')} - 1, t - 1\}\}, \quad t \in \mathcal{T}, \\
& \quad \sum_{i \in \mathcal{C} : r \in \mathcal{R}_i} \psi_{rt}^i \leq 1, \quad \forall r \in \mathcal{R}, \quad t \in \mathcal{T}, \\
& \quad \psi_{rt}^i + \omega_{kt} \leq 1, \quad \forall t \in \mathcal{T}, \quad k \in K, \quad r \in \mathcal{R}_k, \quad i \in \mathcal{I}_{rk} : r \in \mathcal{R}_i, \\
& \quad \psi_{rt}^i - \sum_{k \in K : r \in \mathcal{R}_k, \quad i \in \mathcal{I}_{rk}} \omega_{k,t+h} \leq 0, \quad \forall i \in \mathcal{C}, \quad r \in \mathcal{R}_i, \quad h \in \{0, \ldots, \min\{l_i^r - 1, T - t\}\}, \quad t \in \mathcal{T}, \\
& \quad \sum_{r \in \mathcal{R}_f} \varphi_r^f = 1, \quad \forall f \in \mathcal{F}, \\
& \quad \sum_{f \in \mathcal{F}_i : r \in \mathcal{R}_f, \quad t \geq T_r^f - l_i^r + 1} \varphi_r^f \leq \sum_{r=1}^{t} \psi_{rr}^i \leq \sum_{f \in \mathcal{F}_i : r \in \mathcal{R}_f, \quad t \geq T_r^f} \varphi_r^f, \quad \forall i \in \mathcal{C}, \quad r \in \mathcal{R}_i, \quad t \in \mathcal{T}, \\
& \quad \chi_t - \omega_{kt} + \omega_{k,t-1} \geq 0, \quad \forall k \in K, \quad t \in \mathcal{T} \setminus \{1\},
\end{align*}
\]
Phase II – The “Routing Phase”

Data:

Key decision variables:

- $z^f_{jt} = 1 \iff$ flight $f$ reaches node $j$ by time $t$
Phase II – Summary

• Fix runway assignments and *sequences* from P1
• Ensure routing constraints met
  – Eg taxiway/runway crossings, etc
  – Fix separation
• If capacity sufficiently high, P1 solution optimal
  – flights processed at same time in P2 as P1
  – flights travel along shortest paths, unimpeded
• Else, runway times will differ slightly
  – here, need to ensure configurations respected
• Guaranteed feasible
Phase II Decisions - Routing
A Bound on the Optimality

• Phase II gives a feasible solution to AOOP
• An optimal solution to AOOP has value no better than the Phase I optimum
  – (It is the full problem, without routing constraints)
• This gives us a bound on the quality of our solution
• A large gap would indicate that the Phase I problem was far from feasible
  – Hence our assumption that runways key bottlenecks would be poor
Computational Experience – Aims

• Are our key assumptions valid?
• Is the methodology computationally tractable?
• Would the use of the methodology result in significant benefits in practice?
### Computational Tractability & Bound on the Optimality Gap

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*=1% optimality gap

Using Data from DFW on 11/2/2009
## Computational Tractability & Bound on the Optimality Gap

Using Data from BOS on 9/28/2010

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Comparison of Optimized and Historic Surface Times

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Surface Congestion

DFW Historic

BOS Historic

DFW Optimized

BOS Optimized
Summary

- *Unified and tractable* approach to solve the entire Airport Operations Optimization Problem (AOOP)
  - Runway configuration management
  - Runway sequencing
  - Flight-runway assignment
  - Flight routing
  - Gate-holding
Limitations

• Who are the decision-makers?
  – Can we implement our solution?

• Uncertainty – see Frankovich (2012)

• Nevertheless:
  – Useful tool for measuring airport performance
  – Analysis of airport infrastructure changes
Thank You