A Trajectory Optimization Based Analysis of the 3Di Flight Efficiency Metric

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What is the 3Di Score?

• Created by the Air Navigation Service Provider NATS

• Measures the fuel efficiency of a flight

• In principle, the 3Di score is calculated by comparing a flown trajectory to a theoretical fuel/CO2 optimum trajectory

• Developed by comparing the fuel consumption of 174000 actual trajectories to 3Di optimal (BADA) trajectories

• Regression analysis used to correlate fuel inefficiencies with
  – Excess flight path distance relative to the great circle distance
  – Level flight segments away from the BADA trajectory
What is the 3Di Score?

• Horizontal Inefficiency $\sigma$

$$\sigma = \frac{D_{ACT} - D_{GCD}}{D_{GCD}}$$

- $D_{GCD}$: Great Circle Distance
- $D_{ACT}$: Actual Distance Flown
What is the 3Di Score?

- **Vertical Inefficiency** $\nu_i$

\[
\nu_i = \begin{cases} 
\frac{t_i(L-l_i)}{T_dL} & l_i \leq L \\
0 & l_i > L 
\end{cases}
\]

- $L$  Requested Flight Level
- $l_i$  Level Flight Level below RFL
- $t_i$  Time of level Flight Level below RFL
- $T_d$  Total flight duration

\[
\nu_{CL} = \sum_{CLIMB} \nu_i \\
\nu_{CR} = \sum_{CRUISE} \nu_i \\
\nu_{D} = \sum_{DESCENT} \nu_i
\]

$\nu_{CL}$, $\nu_{CR}$, $\nu_{D}$ are the vertical inefficiency of the climb, cruise and descent phases
What is the 3Di Score?

• The 3Di inefficiency score $\vartheta$ is then determined by combining the horizontal and vertical inefficiencies into an overall inefficiency score

$$\vartheta = a_1 \sigma + a_2 \nu_{CL} + a_3 \nu_{CR} + a_4 \nu_D + a_5 \nu_{CL} \sigma + a_6 \nu_{CR} \sigma + a_7 \nu_D \sigma$$

where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are 3Di score regression coefficients.
Eurocontrol Review of the 3Di Score

• In 2011 the UK CAA sought stakeholder consultation with regard to the 3Di metric

• Eurocontrol highlighted
  – 3Di Optimal trajectories may not be optimal
  – that there is a need for any flight efficiency metric to include inefficiencies related to the choice of the Requested Flight Level

• Goal of the Work
  – Use a trajectory optimisation method to
    • better understand the 3Di score
    • better understand the definition of an optimum trajectory
    • better understand inefficiencies related to the choice of RFL
What is Trajectory Optimisation?

• Trajectory optimization is the process of designing a trajectory that minimizes or maximizes some measure of performance within prescribed constraint boundaries.

• The goal of solving a trajectory optimization problem is essentially the same as solving an optimal control problem.

\[
\begin{align*}
\min_{X \in X, \ U \in U} \quad & J \\
\text{subject to} \quad & \dot{X} = f(X, U, t) \\
& X(0) = X_0 \\
& \Psi(U_f, X_f) \leq 0
\end{align*}
\]

where:

- \( J \)  
  Performance measure

- \( X \)  
  Aircraft states

- \( U \)  
  Aircraft controls

- \( f(X, U, t) \)  
  Dynamics model

- \( X_0 \)  
  Initial state

- \( \Psi(U_f, X_f) \)  
  Terminal state constraint
The Inverse Dynamics Method

For the inverse method, the Cartesian positional states \( r_j \) \((j = 1, 2, 3)\) and their derivatives are parameterised as

\[
\begin{align*}
 r(\tau)_j &= \sum_{k=0}^{7} \frac{a_k \tau_j^k}{\max(1, k(k-1))} \\
r'(\tau)_j &= \sum_{k=1}^{7} \frac{a_k \tau_j^{k-1}}{\max(1, (k-1))} \\
r''(\tau)_j &= \sum_{k=2}^{7} a_k \tau_j^{k-2} \\
r'''(\tau)_j &= \sum_{k=3}^{7} (k-2) a_k \tau_j^{k-3}
\end{align*}
\]

9 optimization variables \( \Xi = [r_{10,f}', r_{20,f}', r_{30,f}', v_{0,f}', \tau_f] \) with “virtual” time \( \tau \)
Inverse Dynamics

- Given

\[
\begin{bmatrix}
    \dot{r}_1 & \dot{r}_2 & \dot{r}_3 \\
    \ddot{r}_1 & \ddot{r}_2 & \ddot{r}_3 \\
    \dddot{r}_1 & \dddot{r}_2 & \dddot{r}_3
\end{bmatrix}
\]

- From dynamic and kinematic equations
  - Remaining states and controls

\[
v_t = \sqrt{\dot{r}_1^2 + \dot{r}_2^2 + \dot{r}_3^2}
\]

\[
\gamma = \arcsin\left(\frac{\dot{r}_3}{v_t}\right)
\]

\[
\chi = \arctan\left(\frac{\dot{r}_2}{\dot{r}_1}\right)
\]

\[
T = m(\dot{v} + g \sin \gamma) + D
\]

\[
n = \sqrt{(v_t \dot{\gamma} + g \cos \gamma)^2 + (v_t \dot{\chi} \cos \gamma)^2}
\]

\[
\phi = \arctan\left(\frac{v_t \dot{\chi} \cos \gamma}{v_t \dot{\gamma} + g \cos \gamma}\right)
\]
Differential Evolution

The IDVD method discretises the infinite dimensional optimal control problem and allows it to be treated as a finite dimensional Non Linear Programming (NLP) problem

$$\min_{X, U} J(X, U), \quad s.t. \quad c(X, U) \leq 0 \quad c \in \mathbb{R}^M$$

- Solved using the stochastic Differential Evolution (DE) NLP method
- Open standard method*
- Useful for nonlinear multi-modal problems

* http://www1.icsi.berkeley.edu/~storn/code.html
Analysing the vertical profile

Aim: Comparisons between the 3Di Optimum and IDVD generated vertical trajectories for fuel consumption

Piecewise polynomials used for IDVD
Climb-cruise-descent scenarios

\[ r(\tau) = \begin{cases} 
    r_{P1}(\tau), & \tau \in [\tau_0, \tau_1] \\
    r_{P2}(\tau), & \tau \in [\tau_1, \tau_2] \\
    r_{P3}(\tau), & \tau \in [\tau_2, \tau_f] 
\end{cases} \]

37 Optimisation variables

\[ \Xi = \begin{bmatrix} 
    x_0''' , y_0''' , z_0''' , v_0'' , \\
    x_1 , y_1 , z_1 , x_1' , y_1' , z_1' , x_1'' , y_1'' , z_1'' , x_1''' , y_1''' , z_1''' , v_1'' , \\
    x_2 , y_2 , z_2 , x_2' , y_2' , z_2' , x_2'' , y_2'' , z_2'' , x_2''' , y_2''' , z_2''' , v_2'' , \\
    x_f , y_f , z_f , v_f , \\
    \tau_1 , \tau_2 , \tau_3 \end{bmatrix} \]

Improved trajectory solutions
Analysing the vertical profile

CLIMB
- A Continuous Climb Departure (CCD) from ground to cruise gives a 3Di score of zero.
- Offering more CCDs and CCDs to higher levels will improve NATS score.
- The climb rate/gradient does not affect the score, only periods of level flight.

Source: 3di Environmental Performance Measure
Analysing the vertical profile

• Continuous Climb Departure with constant acceleration

Source: SESAR and the Environment
Analysing the vertical profile

- Unlike recommended procedures IDVD-DE solution suggests low, level segment, acceleration
- Expensive in terms of low level fuel burn
- But can expedite climb, reducing overall fuel to climb
- 3Di Score ranked the least efficient climb trajectory as the most efficient

Fuel efficient departure climb to a RFL scenario, distance profiles

Height profile
Speed profile
Climb rate profile
Thrust profile
Analysing the vertical profile

• 3Di “Perfect Flight” trial*
  • NATS, British Airways and BAA
  • A321 flight from London to Edinburgh
• Key finding
  • “The Airbus A321 was able to fly without the everyday but necessary constraints imposed on air traffic because it was a one-off. It was also able to fly at its most fuel-efficient altitude for longer than usual”*

• Simulation scenario designed around Perfect Flight trial
  • Compares a 3Di Optimal (BADA) vertical trajectory against a IDVD-DE generated trajectory for a A321 London to Edinburgh scenario

Source: NATS, British Airways and BAA in UK-first with “Perfect Flight”
Analysing the vertical profile

- IDVD-DE reduces fuel consumption relative to the BADA trajectory
  - Faster climb
  - Slower cruise
  - Slower descent

- Unlike flight trial, cruise is shortened to better take advantage of descent L/D ratios

- Shows coupling between climb, cruise and descent phases for short duration flights
Key Findings

• Only using level segments to define vertical flight inefficiency is a little reductive
  • 3Di score not sensitive to flight speed schedule and related fuel inefficiencies

• The importance of the speed schedule
  • significantly impacts the overall energy management of the aircraft, and therefore flight fuel efficiency
  • There is a speed schedule trade-off between the most CO2 efficient trajectory and the user preferred trajectory
    • Minimum CO2 trajectories often have longer flight times due to slower cruise and descent speeds
      – However, may not be user preferred as flight time costs operators money
    • Fast climbs require higher (non de-rated) thrust levels on climb out
      – However, may not be user preferred as potentially increases maintenance costs

• Operators manage flight efficiency through the speed schedule
  • However, the impact of ATM recommended procedures on operators speed schedule rarely considered
Impact of ATM Constraints

Requested Flight Level (RFL) – Flight Level requested in the flight plan

CRUISE

- Giving aircraft their last planned Requested Flight Level (RFL) gives a 3Di score of zero.
- Giving aircraft levels above their RFL also gives a 3Di score of zero, a simplification of the metric. In reality giving aircraft levels above RFL on the pilots' request will usually result in fuel savings.
- The closer to the RFL the better for 3Di.

- The further below the RFL that the aircraft cruises the worse for 3Di.
- The more time spent below RFL the worse for 3Di.
Impact of ATM Constraints

- London to Paris flight - What is the impact of SID-STAR-Airway constraints on fuel efficiency?

Both IDVD-DE generated trajectories

SID-STAR-Airway constraints alter the most efficient Requested Flight Level (RFL)

- As the RFL is an input to the 3Di score calculation
  - Unquantified inefficiency in the 3Di score

London-Paris. Impact of ATM constraints scenario
Impact of ATM Constraints

- Constrained and unconstrained speed profiles
- Constraints limit speed profile management
- Again, higher initial fuel consumption is used to minimise overall fuel consumed

London-Paris. Impact of ATM constraints scenario
Impact of ATM Constraints

3D and time based speed profiles

- Constrained and unconstrained speed profiles

3D and time based fuel burn profiles

- Constraints limit speed profile management
- Constraints prolong flight time, also increasing fuel consumed

Higher initial fuel burn

Constraints prolong flight time – increasing fuel consumption

London-Paris. Impact of ATM constraints scenario
Key Findings

• Impact of ATM Constraints
  • 17% fuel burn difference between constrained and unconstrained trajectory solutions
  
  • ATM related flight fuel inefficiencies due to
    • Track-extension
    • Constrained speed management
    • Constrained RFL

  • ATM related flight fuel inefficiencies not typified by
    • level flight segments
Key Findings

- Impact of ATM Constraints
  - 17% fuel burn difference between constrained and unconstrained trajectory solutions

- ATM related flight fuel inefficiencies due to
  - Track-extension
  - Constrained speed management
  - Constrained RFL

- ATM related flight fuel inefficiencies not typified by
  - Level flight segments

Factors currently contributing to the 3Di score
Environmental Trade-offs

Noise & Emissions measure Pareto trade-off plot

Noise & Emissions Pareto flight path profiles

Noise & Emissions Pareto height profiles

Noise & Emissions Pareto thrust profiles
Examining Our Assumptions

• Trajectory optimisation methods
  • Allows ATM researchers to examine our assumptions regarding what we generally define as an optimal trajectory (be it CO2, or user preferred, etc)

  • Allow the investigation of assumptions regarding trade-offs

  • In the case of the 3Di score, these assumptions are particularly important ones, and have a significant impact on what we can conclude from studies that use the metric

  • Explore the constraints that most limit the chosen efficiency measure
Future Work

• There are a wide number of control based trajectory methods that can be applied to the trajectory optimisation problem
  • The results of the IDVD-DE approach could be confirmed or revised through the use of other, potentially more accurate, methods

• Could optimal planned trajectories be generated to assess the efficiency of every flown trajectory?
  • SESAR goal?
Questions?