Brownian Motion Delay Model for the Integration of Multiple Traffic Management Initiatives

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Motivation

• There is no technology that provides effective support for procedural consideration of the effect of strategic programs on tactical Traffic Management Initiatives (TMIs)

• A key challenge in the design and use of Air Traffic Management (ATM) decision support tools is to determine how much control should be applied to the flow of traffic and at what point in the flow should it be applied:
  – Inefficiencies can be caused by either under or over-control of the flow

• Our research presents a closed-form stochastic analytical solution, including both uncertain and dynamic demand and capacity, which quantifies the interactions between TMIs:
  – We evaluate the probability that one TMI over-/under-controls the flow seen by a down-stream TMI
Brownian Motion Delay Model
Brownian Motion Delay Model

- Fundamental Equation to Model TFM Delay:

\[ Y(t) = \int_{0}^{t} \frac{D(t) - C(t)}{C(t)} \, dt \]

- Stochastic Model Development Based on the TFM Delay Equation
- Add Uncertainty in Model of Demand:

\[ \hat{D}(t) = D(t) + \varepsilon(t) \quad \hat{Y}(t) = \int_{0}^{t} \frac{D(t) + \varepsilon(t) - \hat{C}(t)}{\hat{C}(t)} \, dt \]

Where \( \varepsilon(t) \) is a Gaussian Random Process with Variance \( \sigma^2 \)

\[ \hat{Y}(t) = Y_0 + \left( \frac{D - \hat{C}}{\hat{C}} \right) t + \frac{\sigma}{\hat{C}} W(t) \]

Where \( W(t) \) is a Brownian Motion Process
Brownian Motion Delay Model

• Analytically Derive a PDF of Over-Control and Under-Control
  – Using the Brownian Motion Result
    \[ PDF_Y(t, y)dy = \frac{1}{(\sigma/C)\sqrt{2\pi t}} e^{-\left(y - y_0 - \left(\frac{D-C}{C}\right)t\right)^2 / 2(\sigma/C)^2 t} \]

• Probability of Over-Control is the Likelihood that the Delay is Zero (or Less)
  \[ P(\text{over-control})(t) = \int_{-\infty}^{0} PDF_Y(t, y)dy \]

• Probability of Under-Control is the Likelihood that the Delay Exceeds Some Threshold, \(Y_T\)
  \[ P(\text{under-control})(t) = \int_{Y_T}^{\infty} PDF_Y(t, y)dy \]
General Brownian Motion Model

• Time-Varying Demand and Capacity with uncertainty as a piece-wise constant function:

\[ D(t), \epsilon_D(t), C(t), \epsilon_C(t) = \begin{cases} 
D_0, \epsilon_{D_0}, C_0, \epsilon_{C_0} & : t_0 \leq t < t_1 \\
D_1, \epsilon_{D_1}, C_1, \epsilon_{C_1} & : t_1 \leq t < t_2 \\
... & \\
D_P, \epsilon_{D_P}, C_P, \epsilon_{C_N} & : t_{N-1} \leq t < t_N 
\end{cases} \]

• Generalized Brownian Motion Model:

\[ \hat{Y}(t) = Y_0 + \mu(t) + \sigma(t)W(t) \]

\[ \mu(t) = \sum_{i=1}^{k} \mu_{i-1}(t_i - t_{i-1}) + \mu_k(t - t_k) - t \quad t_k \leq t < t_{k+1} \]

\[ \sigma^2(t) = \sum_{i=1}^{k} \sigma_{i-1}^2(t_i - t_{i-1}) + \sigma_k^2(t - t_k) \quad t_k \leq t < t_{k+1} \]
Empirical Identification of Prediction Errors and Model Calibration
We chose PHL arrivals TBFM data to evaluate the prediction errors of the BM model.

Data was collected from 30 snapshots of the TBFM system at about 30 minutes before an arrival push (2014 data).

In the selected snapshots PHL was operating in West configuration.

The dataset includes 2,349 flights.

Surface data was used to: estimate PHL arrival capacity and to obtain actual flight delays (needed to calculate prediction errors).
The BM model approximates capacity as a set of normally distributed variables.

15-minute arrival counts for 5 months were collected (ASDE-X data).

We selected time intervals in which PHL was operating at full capacity, i.e. demand exceeded capacity.

For the selected data points the capacity is characterized by $\mu = 14.5$, $\sigma = 2.5$, the hourly rate is $[\mu - \sigma, \mu + \sigma] = [50.7, 60.7]$. 
• Given a set of flights with an assumed arrival time distribution, find the equivalent count distribution within a given time interval, denoted by \([ts; te]\).

• Approach:
  – Assume that the time of arrival for a given flight, \(f_k\), is Gaussian with a mean \(\mu_k\) value of and a standard deviation, \(\sigma_k\).
  – Calculate the probability of each flight arriving at the different time intervals.
  – Waring’s theorem gives the probability that exactly \(r\) out of \(n\) possible events should occur
  – Waring’s probabilities at each time step are approximated using a normal distribution, which is fed into our model.
We used the Waring counts methodology, with the following arrival time parameters:

- For **flights in the air**, which have an STA assigned by the TBFM system, we assume **1 minute** as the standard deviation of delivery time error for the flights to the meter fix. This will only apply to flights outside of the meter fix. We also include an additional terminal standard deviation of **2 minutes**, which reduces linearly with ETA to the runway.

- For **flights on the ground**, the standard deviation includes the ``in the air'' standard deviation plus 3 minutes of departure compliance error.
Prediction Errors and Model Calibration

- Prediction error vs time since snapshot:
  
  - Better prediction performance for flights in the air
  - Error distribution for flights on the ground skewed to the right
Prediction Errors and Model Calibration

- Prediction error histograms:

Flight in the air
Symmetric distribution
\( \mu = 2.7 \) min. (excluding outliers)

Flight on the ground
Skewed to the right
\( \mu = 14.3 \) min.
Prediction Errors and Model Calibration

- Prediction error vs predicted values
- Buckets were defined for the predicted delay and sigma
- The actual values are estimated for each bucket using the actual arrival times

-Again, flights on the ground skewed delay distribution lead to actual delays that are higher than predicted delays.
- The actual standard deviation seem to be limited at 3min. The predicted delay $\sigma$ is limited at zero, a lower limit would reduce prediction errors.
• TBFM Snapshots taken every 5 minutes from 17:00z to 21:00z on August 24\textsuperscript{th} 2014
• PHL was in East configuration
• The animation includes:
  – Over-/Under-control probabilities
  – 15-minutes demand counts and capacity mean and sigma
PHL - TBFM Animation

- Graph 1: Probability vs. Time since snapshot (minutes)
  - Under-control, th=5min
  - Over-control

- Graph 2: Rate, 15min step vs. Time since snapshot (minutes)
  - Arrival Demand
  - Capacity +/- sigma
Case Study - GDP-TBFM Interaction
• The goal is to characterize how the implementation of a GDP affects TBFM metering

• Sample Questions:
  – Is the GDP solution over/under-constraining the arrival flow seen by TBFM metering?
  – How is the GDP solution affecting the CFR procedure? Are internal departure finding open en route slots? Are large delays for inbound departures ("double delay") or airborne delay needed?

• 30 days of ADL data (2011, 2012) were processed to characterize the typical behavior
We used ADL data to model both GDP and TBFM demand, but the capacity profiles differ:

- GDP capacity obtained using the arrival rates defined in the ADL data: 30fl/h before fog lifts and 60fl/h after fog lifts
- TBFM capacity is estimated using the published separation matrices and expected flight mix

**Capacity Profiles for July 25\textsuperscript{th} 2012**
We modeled three different cases:

- **Uncontrolled Case**: the arrival demand is generated using ETAs from an ADL snapshot taken right before the GDP parameters were published.

- **Controlled Case**: the arrival demand is generated using ETAs and CTAs available in an ADL snapshot taken when the GPD parameters are first published.

- **Controlled Case – Internal Departures Excluded**: the arrival demand is generated using ETAs and CTAs available in an ADL snapshot taken when the GPD parameters are first published and excluding internal departures. Internal departures are defined as flights which departure airport is less than 300nm away from SFO. This Case is related to the CFR problem.
Uncontrolled Case. GDP

• Results summarize 30 days of data

• **High probability of a 15-minute delay or higher**, indicating that the implementation of the GDP was justified.

• From all the days included, the day with the lowest maximum of the under-control probability was May 27th 2012, with a maximum value of 0.918 - > **High probability of 15-minute delay for all the days included**

• These curved can be used by TMs to identify when is a good time for the GDP to start/end:
  – **180 minutes before the forecast clearing time** is typically a good time to start the GDP program.
Controlled Case. GDP

- The probability of delay reaches its maximum at about 40 minutes before the forecast clearing time.
- The under-control probability drops before the forecast clearing time. This indicates that capacity typically increases sooner than the arrival rate:
  - Traffic managers are using the clearing forecast time plus a buffer to determine when the arrival flow can be increased to normal levels -> the figure depicts a ~50 minutes buffer
Controlled Case. TBFM

- The over-control curve shows that the GDP solution (EDCTs) leads to high over-control and most likely to unused capacity.
- The increasing trend in the under-control probability is caused by:
  - The increase in uncertainty with time, and
  - The fact that the actual TBFM capacity rate after the fog lifts is lower than the rate defined in the ADL data in the GDP context.
- A day with especially high delay, and where the GDP solution did not over-controlled the flow was July 27th 2012:
Controlled Case – Internal Departures Excluded. TBFM

- In this case internal departures (<300nm from SFO) are removed from the demand set.
- On average 21.4% of the departures are internal departures.
- The difference between delay probabilities for the controlled case with departures included and departures excluded indicates how much additional delay airborne flights would need to accommodate if no control action (additional ground delay) is applied to inbound departures.
- As the figures below show, adding/removing internal departures does not have a major impact on the under-/over-control probabilities.
Next Steps

• Detailed characterization of demand and capacity uncertainty parameters.
  – Improvements in model accuracy by differentiating among more flight states and uncertainty levels (e.g. flight on the ground has pushed back or not)
  – Detailed capacity modeling, where the capacity $\mu$ and $\sigma$ vary over time according to the state of the capacity constrained resource.

• Additional modeling of out-time prediction error and other effects (e.g., surface congestion, de-icing) to address skewed delay distributions to better model flights on the ground.

• Further development of downstream propagation of demand uncertainty. For example, there could be a sector traveled by some SFO arrivals, which has a chance of being affected by convective weather and its capacity may be reduced. This phenomenon would add additional uncertainty to SFO arrival demand.
Questions and Answers