Extraction and Interpretation of Geometrical and Topological Properties of Urban Airspace for UAS Operations

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Abstract— With the rapid adoption of operational concepts of Unmanned Aerial Systems (UAS), a large amount of traffic is expected to flow into low-level airspace. However, this low-level airspace contains existing environment of people and surrounding structures that are sensitive to the risk posed by UAS operations. To provide necessary separation and flight planning services, UAS traffic management (UTM) system will need to first identify available airspace that vehicles can operate with an acceptable level of risk. This study attempts to claim that much of what is perceived as empty airspace may not be available for operational use and that the available airspace in highly urbanized areas has a complex geometric form. Geometrical and topological analysis of such complex airspace geometry is necessary as it can provide valuable insights for fully utilizing the finite UTM airspace. In this paper, we present topography map and skeletal graph to interpret underlying geometrical and topological features of urban airspace. Airspace topography map displays a 2D projection of the lowest navigable altitude at specified latitude and longitude, whereas airspace skeletal graph uncovers horizontal- and vertical-connectivity of its components. Both methods not only provide a compact and informative abstraction of airspace but also can be used to partition the entire airspace into different levels based on geometrical and topological properties.

Keywords-UAS traffic management (UTM); airspace availability; airspace topography map; airspace skeletonization; airspace partitioning;

I. INTRODUCTION

With the rapid adoption of the operational concept of unmanned aerial systems (UAS), a large amount of traffic is expected to flow into low-level airspace [1-3]. Several air navigation service providers (ANSPs) have initiated the development of UAS traffic management (UTM) system to accommodate potential UAS traffic [4-9], but there still remain challenges in identifying available airspace that unmanned aircraft (UA) can fly within an acceptable level of risk.

Low-level airspace contains existing environment of people and surrounding structures that are sensitive to the risks of UAS operations. In such environment, much of what is perceived as vacant airspace may not be available for operational use. In many literatures, however, UTM airspace is often regarded nearly free of obstacles, and the geospatial complexity arising from geometric distribution of surface obstacles is not fully addressed [10-14]. Our previous study was the first attempt highlighting the importance of incorporating geospatial elements in assessing airspace availability [15]. By applying two types of geofencing to account for safety clearances, a notable difference was found between free airspace as seen by the human eye and available airspace. One of the major findings was that the resulting airspace has a complex geometric shape, but important spatial features such as horizontal and vertical connectivity have not been investigated. Such research effort is needed because the geometric structure of available airspace will impact the complexity of flight paths and operation volume arrangements.

In this study, we introduce a topological approach to extract and interpret the spatial structure and characteristics of low-level airspace. We first introduce a topographic map of airspace to provide a quantitative representation of vertical and horizontal dimension of available airspace. Second, a topological skeleton of airspace is extracted to uncover underlying connectivity of its component parts. Furthermore, using the skeletal abstraction of entire airspace, we partition the urban airspace into four different levels according to geometric and topological properties.

The rest of the paper is organized as follows. Section 2 explains the airspace availability assessment framework. Geometrical and topological assessment methodology is presented and discussed in Section 3, and the case study results for the actual urban environment are presented in Section 4. Section 5 provides conclusions and future study ideas.

II. AIRSPACE AVAILABILITY ASSESSMENT FRAMEWORK

An important first step to extract geometric and topological properties of airspace is to identify available airspace. In this study, we adopt and improve the airspace availability assessment framework introduced in our previous work [15]. We apply two types of geofencing to incorporate vehicle operational requirements as well as provide a protection boundary to surrounding environments. Notable modifications
to data processing have been made to handle large-scale obstacle data. This section describes terminologies associated with airspace availability, and details the modified data processing steps.

A. Available airspace and geofencing

1) Terminologies: Available airspace can be categorized into free and usable airspace, as defined in [15]. Free airspace is space with no surface obstacle. It is equivalent to the empty space seen by the human eye. Usable airspace is a space that a spherical containment limit of UA can fit in. It is the space after removing geofenced space from free airspace (Fig. 1). Usable airspace is identified by applying keep-out and keep-in geofence. Keep-out geofence is used to put an artificial fence around designated areas, so that vehicles stay out from the fenced boundary [16-18], while keep-in geofence concerns vehicle’s safety margin in relation to potential flight deviations caused by mechanical instability or navigational inaccuracy [19-22]. UA is not treated as a point but as a sphere containing the aircraft, which is in its essence keep-in geofence (Fig. 1).

2) Geofencing implementation: Keep-out geofence is modeled as a buffer space around static obstacles such as buildings, whereas keep-in geofence is modeled using alpha shape method (see Section II.B). Boundary points of obstruct polygons are used as input data for geofence implementation. Since discretizing the entire airspace into uniform grids creates an unnecessary computational load, we chose to use the boundary points used to handle large-scale (i.e. citywide) obstacle data. In preprocessing, a set of vertices that define an obstacle polygon is extracted, and a buffer of size $\delta$ is generated on each vertex of the obstacle polygon to apply keep-out geofencing. For keep-in geofencing, we constructed alpha shapes of radius $r$ on sampled boundary points. The output is a triangulated mesh representing airspace not usable by sUAV of keep-in radius $r$.

B. Alpha shape method

Alpha shape is a computational geometric technique used to construct a shape of a finite point set using a spherical ball or $\alpha$-ball [23-26]. In this subsection, we provide an intuitive description of alpha shape and explain how it is applied in the airspace availability assessment.

1) Intuitive description of alpha shape: Consider there is a point set $S$ in $\mathbb{R}^3$ and a real number $\alpha$ such that $0 \leq \alpha \leq \infty$. Imagine $\mathbb{R}^3$ as a 3D space and $S$ as a set of obstacle points. Suppose there is a spherical eraser of radius $\alpha$ (i.e. $\alpha$-ball) that removes the 3D space with condition not to touch any obstacle points in $S$. The remaining space after removal will consist of curved boundaries enclosing $S$. The alpha shape of $S$ is then obtained by replacing the curves with straight lines or planes. For mathematical definition of alpha shape, please refer to [23-26].

2) Application of alpha shape to airspace availability assessment: The key idea of applying the alpha shape method to the airspace availability assessment is to relate $\alpha$-ball with the vehicle’s spherical containment limit [15]. Note once again that a vehicle is not treated as a point but assumed as a sphere containing it, as shown in Fig. 2a. Eliminating where $\alpha$-ball cannot fit in will result in airspace usable by vehicle with spherical containment limit of a specific size. By controlling the size of $\alpha$-ball, one can observe that some portion of free airspace cannot accommodate vehicles with a specific keep-in radius, as depicted in Fig. 2b.

III. GEOMETRICAL AND TOPOLOGICAL ASSESSMENT METHODOLOGY

To enhance the understanding of low-level airspace composed of various horizontally- and vertically-connected components, we introduce methodologies that generate a topography map and extract a skeletal graph from airspace availability information.

A. Airspace topography map

Airspace topography map shows a 2D projection of the lowest navigable altitude at specified latitude and longitude. A sample airspace topography map of a built-up area in San Francisco in Fig. 4, where the lowest navigable altitude per unit
cell is visualized in grayscale at resolution of 3m by 3m. The darker the cell is, the higher the airspace is occupied by geographic obstacles and geofences. A contour line connecting spaces of equal minimum navigable altitude is also be overlaid in the map, as shown in Fig. 4.

B. Airspace skeletonization

To capture how airspace segments are connected to each other to form the whole, we extract a skeletal graph of usable airspace based on a topological concept called Reeb graph. Reeb graph is a shape descriptor widely used in computational geometry and computer graphics applications such as 3D shape encoding, segmentation, and comparison [28-35]. In this study, we derive an approximated Reeb graph \( \mathcal{G} \in \mathbb{R}^3 \) from usable airspace \( A \in \mathbb{R}^3 \) using Mapper algorithm [36,37]. Topological features of airspace is described by connectivity of the graph, providing a structural overview of an entire airspace. The following subsections introduce necessary topological notions of Reeb graph and implementation details of Mapper algorithm.

1) Formal definition of Reeb graph: Let \( f : X \rightarrow \mathbb{R} \) be a continuous function defined on a domain \( X \). For each scalar value \( a \in \mathbb{R}, \) the level set \( f^{-1}(a) = \{ x \in X | f(x) = a \} \) may have multiple connected components. Reeb graph of \( f \), denoted by \( \mathcal{R}_f(X) \), is obtained by reducing each connected component in a level set to a single point. \( \mathcal{R}_f(X) \) is the image of a mapping function \( \Phi : X \rightarrow \mathcal{R}_f(X) \) where \( \Phi(x) = \Phi(y) \) if and only if \( x \) and \( y \) belong to the same connected component in a level set of \( f \) (Fig. 5). As the value \( a \) increases, connected components in the level set \( f^{-1}(a) \) appear, disappear, split and merge, and Reeb graph of \( f \) tracks such changes.

2) Approximated Reeb graph computation using Mapper algorithm: The best known algorithms of computing exact Reeb graph from a 3D mesh structure require expensive temporary data structures [34,35]. In this study, we compute approximated Reeb graph from a point set [36,37]. Specifically, we adopt a clustering-based graph construction algorithm called Mapper to compute an approximation of Reeb graph. Mapper algorithm is partial clustering of overlapping data subsets [45]. The basic idea is to divide the original data into overlapping subsets, apply a clustering algorithm to each subset, and interpret the connectivity among those partial clusters. Any partial clusters are ‘connected’ if there exist any non-empty intersections. By representing each cluster as a node and each connection as an edge, a graph-like structure is derived as shown in Fig. 6.

To apply Mapper algorithm to extract a skeletal graph, we first sampled points of usable airspace and divide the sampled points into overlapping subsets based on altitude. Suppose an altitude function \( h : X \rightarrow Z \) for a point set \( X \in \mathbb{R}^3 \) and \( z \) coordinate \( Z \in \mathbb{R} \). \( Z \) is divided into a set of overlapping intervals \( \mathcal{U} = \{ U_k \} \), so that \( X \) is decomposed into subsets \( h^{-1}(U_k) = \{ h^{-1}(U_k) \} \). For each subset \( h^{-1}(U_k) \), we apply the density-based clustering algorithm, DBSCAN, to divide \( h^{-1}(U_k) \) into a set of disjoint partial clusters. Note that each partial cluster is a path-connected space, meaning that for any two points in the partial cluster there exists a path between them. Each partial cluster is represented as a node, and the nodes are connected by an edge if any clusters have points in common. In this study, the length of interval is 10 meters, and the overlapping portion between successive intervals is 50%.

C. Airspace partitioning

Abstraction of airspace into a compressed graph-like structure allows us to infer, analyze, and interpret spatial characteristics such as connectivity. This low-dimensional skeletal representation, instead of bulks of grids, point clouds or triangulated surfaces, is simple and practical not only in visualizing the connectivity and but also in partitioning airspace into multiple segments based on geometrical and topological properties.

1) Skeletal graph decomposition: Given a skeletal graph \( G \), let \( v \) represent nodes of \( G \) and \( h(v) \) indicate the altitude of \( v \). Note that each node in a skeletal graph represents a path-connected space, where a path exists between any two points inside the space. We classify each node into terminal, middle, and branch node, where terminal nodes \( v^T \) is of degree 1,
middle nodes $v^M$ of degree 2, and branch nodes $v^B$ of degree 3. Each terminal node indicates an airspace segment available from the ground level. As altitude increases, terminal nodes gradually develop and eventually merge into others at branch nodes. The evolution of terminal nodes is summarized by information at which altitude it is first generated (birth-altitude) and is finally merged to another (death-altitude).

The skeletal graph $G$ is then decomposed into a group of stems, where each stem is defined by terminal nodes and branch nodes. Suppose nodes $v_i, v_j \in v^T \cup v^B$ such that $h(v_i) < h(v_j)$. We define that $v_i$ and $v_j$ form a stem $S(v_i, v_j)$ if the two nodes are connected via $v_{i_1}, \ldots, v_{i_n} \in v^M$ such that $h(v_{i_1}) < h(v_{i_2}) < \cdots < h(v_{i_n}) < h(v_j)$ for $n \geq 1$. For $S(v_i, v_j)$, we name $h(v_i)$ as birth-altitude and $h(v_j)$ as death-altitude. Then, each stem is summarized as a point in birth-death diagram. For example, a skeletal graph $G$ in Fig. 7a can be decomposed into a set of stems: $S(v_1, v_3), S(v_3, v_4), S(v_2, v_5), S(v_4, v_5)$, and $S(v_5, v_6)$. The birth- and death-altitude of $S(v_2, v_4)$ are $h(v_2)$ and $h(v_4)$, respectively, and shown accordingly in birth-death diagram.

2) Airspace partitioning logic: In this paper, airspace is divided into four levels based on two factors: proximity to ground level and relative volume (see Fig. 7b). Specifically, we first divide stems into two groups, by identifying whether airspace segment is available from ground level. Then, we compare the relative volume of airspace segments. Resulting airspace partitions are color-coded as shown in Fig. 7b. Characteristics and potential use of the four-level airspace are described below.

a) Level 1: This level of airspace is characterized by relatively low volume compared to Level 2 airspace and proximity to ground level. Operational use may be limited to ascent/descent operations for a small number of traffic.

b) Level 2: This level of airspace is characterized by relatively high volume compared to Level 1 airspace and proximity to ground level. It can best be utilized for ascent/descent operations and may require demand/capacity management at intrasegment level.

c) Level 3: This level of airspace is characterized by relatively low volume compared to Level 4 airspace and intermediacy in terms of situated altitude. It may be utilized to transit traffic to Level 4 airspace from Level 1 or 2 airspace. Demand/capacity management may be required at intersegment level.

d) Level 4: This level of airspace is characterized by relatively high volume compared to Level 3 airspace and remoteness from ground level. It may be used mainly for cruising between distant locations (e.g. air highway).

Figure 7. Skeletal graph decomposition and summarization: (a) birth-death diagram; (b) partitioning logic

Figure 8. San Francisco case study area: a) building distribution; b) sample airspace topography map
IV. CASE STUDY RESULTS AND DISCUSSIONS

We present our case study results of San Francisco airspace in this section. Note that we divided the city area into multiple parts and selected two areas of distinct geometries – D31 and D35 (see Fig. 8). Using the building distribution data, we generated airspace topography maps and skeletal graphs with respect to specific geofence parameter combinations. Altitude range is up to 150 meters above ground level.

A. Airspace topography map of case study areas

Fig. 9 shows airspace topography map of D31 and D35 with respect to geofence combinations of \((\delta, r) = (30, 10)\) and \((30, 30)\). In the map, the lowest altitude that sUAV can fly is represented in grayscale at resolution of 3m by 3m unit cell. The darker the map is, the higher the flight altitude should be to avoid geographic obstacles and geofences. One can easily see that D35 airspace is filled with darker areas than D31 in both geofence combinations. More importantly, there is a high variability of brightness (i.e. minimum navigable altitude) in D35. It implies that there exist peaks and ridges in D35 that vehicles need to either fly over or detour in order to pass such regions. Also, there are U-shaped valleys or conical-shaped holes of varying depths, where vehicles need to climb up or down. On the contrary, D31 airspace is barely influenced by static obstacle or geofencing, resulting in a low variability of brightness in the map. It is inferred from the map that D31 airspace is mostly open at above around 40 meters.

When airspace is more conservatively configured with a larger keep-in parameter, the overall topography changes as depicted in Fig. 9. By the effect of increased \(\alpha\)-ball radius from 10 to 30 meters in D35 airspace, some portion of aerial valleys and holes that are not wide enough to fit in a vehicle with a larger spherical containment limit is blocked. Ridges are formed between adjacent peaks, blocking some airways that were originally usable when \(r=10\). On the other hand, D31 airspace is only marginally affected by the increased keep-in parameter due to its relatively dispersed distribution of geographic obstacles.

B. Skeletal graphs and Birth-Death diagram

Now, we present skeletal graphs and birth-death diagrams of D31 and D35 airspace with respect to geofence combination \((\delta, r) = (30, 30)\) in Fig.10 and Fig.11, respectively. The skeletal graphs visualize the vertical connectivity of disjoint airspace segments, while the birth-death diagrams summarize the structure of skeletal graphs. Each point in a birth-death diagram is positioned based on the birth- and death- altitude of a stem, where each point is color-coded according to airspace level. For example, \((20,30)\) represents a path-connected airspace segment that has its minimum navigable altitude of 20 meters and merges to other segment at 50 meter altitude.

The birth-death diagram of D31 shows that its skeletal graph has fewer stems compared to D35, where all nodes are merged.

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**Figure 9.** Airspace topography map of D31 and D35 with respect to two geofence combinations \((\delta, r) = (30, 10)\) and \((30, 30)\).

**Figure 10.** Illustration of D31 airspace: (a) skeletal graph, (b) birth-death diagram, and (c) color-coded airspace partitions.

**Figure 11.** Illustration of D35 airspace: (a) skeletal graph, (b) birth-death diagram, and (c) color-coded airspace partitions.
to a singular node at altitude of 35 meters. In contrast, that of D35 shows this airspace is characterized by complex merging of disjoint segments at various altitudes.

C. Airspace partitioning results
Fig. 10c and Fig. 11c show the results of airspace partitioning in D31 and D35 airspace. Each level of airspace is color-coded according to the partitioning logic explained in Section III.C. Level 1 airspace (in yellow) indicates an available airspace segment of small volume that is close to ground level. The use of this airspace will be largely affected by operational environmental factors such as population density and surrounding sensitive structures because of its proximity to the surface. Its shape is usually a shallow hole, and its utility may be optimized for ascent and descent operations of a small number of traffic. On the contrary, Level 4 airspace (in blue) is by far the largest airspace segment that other segments finally merge into. It is located above 35 meters for D31 and 120 meters for D35. This airspace is mostly obstacle-free and can be used as ‘air highway’ for travelling between distant locations.

Level 2 airspace (in orange) has relatively large volume compared to Level 1. Its shape is usually a deep and wide valley. It may best be utilized for ascent/descent operations and require demand/capacity management at intra-segment level. Level 3 partition (in green) is an intermediate airspace located in between Level 1 or 2 and Level 4, and thus traffic transiting between Level 1 or 2 and Level 4 will frequently occur. Operations departing and landing from/to Level 3 can be managed at a local level. Thus, capacity management would be needed at both intra- and inter- segment level. In D31 airspace, there are three disjoint Level 3 airspace segments at altitudes from 20 to 30 meters. In D35 airspace, there are sixteen Level 3 airspace clusters located at various altitudes ranging from 35 to 115 meters, implying that D35 airspace has a highly complex geometric structure characterized by dynamic merging of airspace segments.

V. CONCLUSION AND FURTHER STUDIES
This paper introduced methodologies to uncover geometrical and topological properties of urban airspace. The proposed approaches of topography map and skeletal graph capture not only geometrical properties of urban airspace landscape but also topological properties of how airspace segments are connected to each other. Both approaches provide a graphical visualization of available airspace, can be readily modified when there is any change in operational environment, and are applicable to any region of any size. Moreover, skeletal representation, instead of bulks of grids, point clouds, or tessellated surfaces, yields a compact and informative abstraction of the entire airspace, and one can utilize this skeletal structure for dividing airspace into smaller partitions. If any airspace management strategies such as scheduling and slot allocation are to be adopted in UTM environment, subdivision of airspace into smaller manageable partitions may add flexibility in managing airspace. Although only the surface obstruction is considered in this study, the resulting topographic map and skeletal structure can be used as a baseline and can always be updated in accordance with any new flight restrictions. This research is also in line with the concept of dividing Class G airspace into three types as proposed by SESAR [6] and the need for modifying current airspace classes as proposed by ICAO [9].

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**BIographies**

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