Automation for Separation with CDOs: Dynamic Aircraft Arrival Routes

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Motivation
✓ Air transportation grows: pros and cons
✓ Increased complexity and environmental effects
✓ Terminal Maneuvering Areas (TMAs) - most congested
✓ Optimization of arrival and departure procedures is needed
✓ **Our solution:**
  - Automatically separated arrivals to reduce complexity and ATCO’s workload
  - **CDOs (Continuous Descent Operations):** promising solution to mitigate environmental effects, according to ICAO and EUROCONTROL: “CDOs allow aircraft to follow a flexible, optimum flight path that delivers major environmental and economic benefits—reduced fuel burn, gaseous emissions, noise and fuel costs—without any adverse effect on safety”
CDOs

✓ CDOs have shown important environmental benefits w.r.t. conventional (step-down) approaches in TMAs
Previous Work

✓ **LiU-LFV**: optimal STARs + time-separated demand-weighted arrival routes (dynamic, for pre-tactical planning)

✓ **UPC**: CDO-enabled optimized arrival procedures (engine-idle, low noise)

**New**: automated time-separated demand-weighted CDO-enabled optimized arrival routes
✓ Location and direction of the airport runway
✓ Locations of the entry points to the TMA
✓ Aircraft arrival times at the entry points for a fixed time period
✓ Cruise conditions (altitude, true airspeed, distance to entry point + path distance inside TMA) and aircraft type for CDO profile generation
Optimal arrival tree that merges traffic from the entries to the runway ensuring safe aircraft separation for the given time period.
Output
Optimal arrival tree that merges traffic from the entries to the runway ensuring safe aircraft separation for the given time period

= a set of time-separated CDO-enabled aircraft trajectories optimized w.r.t. the traffic demand during the given period
Grid-Based MIP Formulation

✓ Square grid in the TMA
✓ Snap locations of the entry points and the runway into the grid
✓ Grid cell side of the length $l$ (separation parameter)
Grid-Based MIP Formulation

- Square grid in the TMA
- Snap locations of the entry points and the runway into the grid
- Grid cell side of the length $l$ ($separation\ parameter$)
- Every node connected to its 8 neighbours
Grid-Based MIP Formulation

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- Snap locations of the entry points and the runway into the grid
- Grid cell side of the length $l$ (separation parameter)
- Every node connected to its 8 neighbours
- Problem formulated as MIP

Based on flow MIP formulation for Steiner trees
Operational Requirements

✓ No more than two routes merge at a point
✓ Merge point separation
✓ No sharp turns
✓ Temporal separation of all aircraft along the routes
✓ All aircraft fly energy-neutral CDO: idle thrust, no speed brakes (noise avoidance)
✓ Smooth transition between consecutive trees when switching
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MIP Formulation

VARIABLES

\( x_e \) - decision variable - indicates whether edge \( e \) participates in arrival tree

\( f_e \) - gives the flow on edge \( e = (i, j) \), non-negative

OBJECTIVES

Total path length: \( \min \sum_{e \in E} \ell_e f_e \)  \( \Rightarrow \) \( \min \beta \sum_{e \in E} \ell_e x_e + (1 - \beta) \sum_{e \in E} \ell_e f_e \)

Total tree weight: \( \min \sum_{e \in E} \ell_e x_e \)
MIP formulation: Constraints

- Flow constraints
- Degree constraints
- Turn angle constraints
- Auxiliary constraints to prevent crossings
- Temporal separation of all aircraft along the routes
- Realistic CDO speed profiles
- Consistency between trees of different time periods
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MIP formulation: Constraints

✓ RE: Flow constraints

\[ \sum_{k:(k,i) \in E} f_{ki} - \sum_{j:(i,j) \in E} f_{ij} = \begin{cases} \sum_{b \in \mathcal{P}} w_b & i = r \\ -w_i & i \in \mathcal{P} \\ 0 & i \in V \setminus \{\mathcal{P} \cup r\} \end{cases} \]

, where \( \mathcal{P} \) set of entry points
\( w_b \) - number of a/c entering TMA from the entry point \( b \in \mathcal{P} \)

\[ x_e \geq \frac{f_e}{Q} \quad \forall e \in E \]
\[ f_e \geq 0 \quad \forall e \in E \quad \text{where } Q \text{ is a large number (e.g., } Q = |\mathcal{P}|) \]
\[ x_e \in \{0, 1\} \quad \forall e \in E \]

MIP formulation: Constraints

✓ **RE: Degree constraints**

\[
\sum_{k:(k,i)\in E} x_{ki} \leq 2 \quad \forall i \in V \setminus \{P \cup r\}
\]

- maximum indegree

\[
\sum_{j:(i,j)\in E} x_{ij} \leq 1 \quad \forall i \in V \setminus \{P \cup r\}
\]

- maximum outdegree

\[
\sum_{k:(k,r)\in E} x_{kr} = 1
\]

- runway \( r \) has only 1 in-going edge

\[
\sum_{j:(i,j)\in E} x_{ij} = 1 \quad \forall i \in P
\]

- only 1 out-going edge for entry points

RE: Turn angle constraint

For each edge $e = (i, j)$ used in the arrival tree, all outgoing edges at $j$ must form an angle of at least $\alpha$ with $e$. The constraint is:

$$c_e x_e + \sum_{f \in \Gamma_e} x_f \leq c_e \quad \forall e \in E$$
MIP formulation: Constraints

✓ **RE: Auxiliary Constraints to Prevent Crossings**

For all points except last column, last row, entries and rwy:

\[
x_{i,i+1,n} + x_{i+1,n,i} + x_{i+n,i+1} + x_{i+1,i+n} \leq 1
\forall i \in V' \setminus \{P \cup r\} : i + 1 + n, i + n, i + 1 \notin \{P \cup r\}
V' = V \setminus \text{last row} \setminus \text{last column}
\]

For different entry point locations:

\[
x_{i,i+1,n} + x_{i+n,i+1} + x_{i+1,i+n} \leq 1 \forall i \in P
x_{i,i+1,n} + x_{i+1+n,i} + x_{i+1,i+n} \leq 1 \forall i + 1 \in P
x_{i,i+1,n} + x_{i+n+1,i} + x_{i+n,i+1} \leq 1 \forall i : i + n \in P
x_{i+1+n,i} + x_{i+n,i+1} + x_{i+1,i+n} \leq 1 \forall i : i + n + 1 \in P
\]

MIP formulation: Constraints

✓ RE: Temporal Aircraft Separation

More variables: \( y_{a,j,t} \) - binary, shows a/c \( a \) at node \( j \) at time \( t \)
\( x_{e,b} \) - binary: edge \( e \) in the route from entry point \( b \)

Connect to \( x_e \):
\[
x_{e,b} \leq x_e \quad \forall b \in \mathcal{P}, \forall e \in E
\]

Set:
\[
y_{a,b,t} = 1 \quad \forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b
\]
\[
y_{a,b,t} = 0 \quad \forall b \in \mathcal{P}, \forall a \in \mathcal{A} \setminus \mathcal{A}_b, \forall t \in T
\]
\[
y_{a,b,t} = 0 \quad \forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b, \forall t \in T \setminus \{t^b_a\}
\]

Forward the information on the times at which \( a \) arrives at nodes along the route from \( b \) to the rwy
\[
\sum_{j: (j,k) \in E} x_{(j,k),b} \times y_{a,j,t} = y_{a,k,t+u}
\]
\[
\forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b, \forall k \in V \setminus \mathcal{P}, \forall t \in \{0, \ldots, T - u\}
\]
Not linear \( \implies \) linearize …

Time separation:
\[
\sum_{\tau=t}^{t+\sigma-1} \sum_{a \in \mathcal{A}} y_{a,j,\tau} \leq 1 \quad \forall j \in V, \forall t \in \{0, \ldots, T - \sigma + 1\}
\]
\( \sigma \) - separation parameter

MIP formulation: Constraints

✓ Flow constraints
✓ Degree constraints
✓ Turn angle constraints
✓ Auxiliary constraints to prevent crossings
✓ Temporal separation of all aircraft along the routes
✓ Realistic CDO speed profiles
✓ Consistency between trees of different time periods
MIP Formulation: Realistic CDO Speed Profiles

✓ The state vector $\mathbf{x}$ represents the fixed initial conditions of the aircraft: TAS $v$, altitude $h$ and distance to go $s$
✓ To achieve environmentally friendly trajectories, idle thrust is assumed and speed-brakes use is not allowed throughout the descent $\rightarrow$ energy-neutral CDO
✓ The flight path angle is the only control variable in this problem $\rightarrow$ control vector $\mathbf{u}$

\[
\mathbf{x} = [v, h, s] \\
\mathbf{u} = [\gamma]
\]
MIP Formulation: Realistic CDO Speed Profiles

✓ A point-mass representation of the aircraft reduced to a “gamma-command” is considered, where vertical equilibrium is assumed → **Dynamic constraints** $f$

✓ **Path constraints** $h$ are enforced to ensure that the aircraft airspeed remains within operational limits, and that the maximum and minimum descent gradients are not exceeded

✓ **Terminal constraints** $\psi$ fix the final states vector

**Dynamic constraints**

$$f = \begin{bmatrix} \dot{v} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} \frac{T_{idle} - D}{m} - g\gamma \\ v\gamma \\ v + w \end{bmatrix}$$

**Path constraints**

$$h = \begin{bmatrix} v_{CAS,min} - v_{CAS} \\ v_{CAS} - VMO \\ M - MMO \\ \gamma_{min} - \gamma \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

**Terminal constraints**

$$\psi = \begin{bmatrix} v - v_f \\ h - h_f \\ s - s_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
The trajectory is divided in two phases: the latter part of the cruise phase prior the top of descent (TOD) and the idle descent.

The original cruise speed is not modified after the optimization process, so the two-phases optimal control problem can be converted into a single-phase optimal control problem.

BADA V4 is used to model the aircraft performance.

\[
J = \frac{f}{v_{\text{cruise}}} + \int_{t_0}^{t_f} (f_{\text{idle}} + C\ell) dt
\]
MIP formulation: Constraints

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NEW: Integration of CDO-enabled Realistic Speed Profiles

Substitute: $y_{a,j,t} \text{ with } y_{a,j,p,n,t} - \text{ binary, indicates whether a/c } a \text{ using speed profile } p \text{ occupies the } n\text{-th vertex } j \text{ at time } t$.

and the corresponding equations with

\[
\sum_{p \in S(a)} y_{a,b,p,1,t_a^b} = 1 \quad \forall b \in P, \forall a \in A_b
\]

\[
y_{a,b,p,k,t_a^b} = 0 \quad \forall k \neq 1 \in L
\]

\[
y_{a,b,p,1,t} = 0 \quad \forall b \in P, \forall a \in A_b, \forall p \in S(a)
\]

\[
y_{a,b',p,k,t} = 0 \quad \forall b' \neq b \in P, \forall a \in A_b, \forall p \in S(a)
\]

\[
y_{a,b,p,1,t_a^b} = 0 \quad \forall b \in P, \forall a \in A_b, \forall p \in S(a)
\]

\[
y_{a,j,p,k,t} \leq \sum_{i \in V; (i,j) \in E} x_{(i,j)} \quad \forall j \in V \setminus P, \forall a \in A, \forall p \in S(a)
\]

Compute $l(b) - \text{ path from } b \text{ to the rwy}$

\[
l(b) = \sum_{(i,j) \in E} x(i,j),b
\]

For each a/c a arriving from b we pick the speed profile from S(a) that has the length $l(b)$:

\[
y_{a,b,l(b),1,t_a^b} = 1 \text{ and } y_{a,b,p,1,t_a^b} = 0 \forall p \neq l(b)
\]

$l(b) \text{ var, not a parameter } \leftrightarrow \text{ aux vars and constraints}$

Separation constraint:

\[
\sum_{\tau=t}^{t+\sigma-1} \sum_{a \in A} \sum_{p \in S(a)} \sum_{k \in L} y_{a,j,p,k,\tau} \leq 1 \quad \forall j \in V,
\]

\[
\sigma - \text{ separation parameter}
\]
MIP formulation: Constraints

NEW: Consistency between trees of consecutive time periods

Define: $x_{ij}$ and $x_{ij}^{old}$ - edge indicators for current and previous periods

$U$ - limits the number of differing edges

$$a x_{ij} \leq x_{ij} - x_{ij}^{old} \ \forall (j, i) \in E$$

$$a x_{ij} \leq x_{ij}^{old} - x_{ij} \ \forall (j, i) \in E$$

$$\sum_{(i,j) \in E} a x_{ij} \leq U$$
Experimental Study: Stockholm Arlanda Airport

✓ Data: Stockholm Arlanda airport arrivals during one hour of operation
✓ Source: EUROCONTROL DDR2, BADA 4
✓ High-traffic scenario on October 3, 2017, time: 15:00 - 16:00
✓ Solved using GUROBI
✓ Run on a powerful Tetralith server, provided by SNIC, LIU: Intel HNS2600BPB nodes with 32 CPU cores and 384 GiB RAM
CDO speed profiles inside TMA

✓ Cruise conditions are obtained from DDR2

✓ TOD position and descent phase are optimized

✓ Same time at the entry point for different path lengths inside TMA
CDO speed profiles inside TMA

- A set of realistic alternative speed profiles for different possible route lengths inside TMA
- Generated for all a/c types arriving to Arlanda during the given period
- Used as input to MIP

Example of A320 speed profiles for different path lengths inside TMA
Results: Stockholm Arlanda Airport

Tree 1: time: 15:00 - 15:30 (10 a/c)

Tree 2: time: 15:30 - 16:00 (7 a/c)
Results: Stockholm Arlanda Airport

✓ **Tree 1:** time: 15:00 - 15:30 (10 a/c)
✓ **Tree 2:** time: 15:30 - 16:00 (7 a/c)
✓ Optimized for 30 min intervals (longer periods may be sub-optimal. Note: time within TMA 5-18 min)
✓ $U = 23$ provides consistency between the trees
✓ Separation: 2 min, ~6 nm
✓ 17 out of 22 arrivals scheduled
✓ 5 filtered out, because of:
  - Initial violation of separation at entry points
  - Potential overtaking problem
  - In general, about 10-15% are not scheduled
Comparison against historical trajectories (Open Sky Network)
## Time Schedule

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Entry point</th>
<th>Simulated time [min]</th>
<th>Entry</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
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<tbody>
<tr>
<td>a1</td>
<td>Ent1 (North)</td>
<td></td>
<td>3</td>
<td>9</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>a2</td>
<td>Ent2 (West)</td>
<td></td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>a3</td>
<td>Ent3 (East)</td>
<td></td>
<td>13</td>
<td>15</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>a4</td>
<td>Ent4 (South)</td>
<td></td>
<td>4</td>
<td>-</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>a5</td>
<td>Ent4</td>
<td></td>
<td>18</td>
<td>-</td>
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<td>19</td>
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<td>Ent2</td>
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<td>57</td>
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t = 15:07
t = 15:08
t = 15:09
$t = 15:10$
t = 15:11
t = 15:12
\[ t = 15:13 \]
t = 15:14
t = 15:15
$t = 15:16$
$t = 15:17$
$t = 15:18$
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$t = 15:21$
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$t = 15:28$
$t = 15:29$
$t = 15:30$
t = 15:30
t = 15:30
t = 15:30
t = 15:31
t = 15:32
Conclusions

✓ Flexible optimization framework for dynamic route planning inside TMA
✓ Automated space and time separation
✓ Environmentally-friendly speed profiles (CDO)
✓ Applicable to any other realistic speed profiles
✓ May be used for TMA capacity evaluation
Future Work

✓ Account for uncertainties due to variations in arrival times
✓ Solve overtaking problem (allow non-optimal profiles, or route stretching)
✓ Consider fleet diversity
✓ Elaborate on implementation possibilities, link to the future operational enablers (data likes, technologies) for air-ground synchronisation (EPP)
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Thank you!

Questions?