Direct Modeling of Flight Time Uncertainty as a Function of Flight Condition and Weather Forecast

Noboru Takeichi and Taiki Yamada
Department of Aeronautics and Astronautics
Tokyo Metropolitan University

13th ATM R&D Seminar
Vienna, Austria, Jun. 20, 2019
Long-term visions for future air traffic management systems
• Introduction of time-based operations to cope with the increase of air transportation demand is planned worldwide.
• Operational time uncertainty must be appropriately evaluated.
• “Flight Time Uncertainty”
  • Inevitably increases as flight progresses
  • Results in arrival time error at waypoints
    • Similar to the longitudinal trajectory uncertainty
Preceding studies on longitudinal trajectory uncertainty

- STD of longitudinal position error is proportional to distance/time.
  - Paielli et al., 1997, Irvine, 2002, etc.
- Validated by data analyses
  - Paielli et al., 1998, Gaydos et al., 2012, etc.

Static model of trajectory uncertainty has been widely applied in the field of ATM research.

Paielli et al., 1997

Paielli et al., 1998

Gaydos et al., 2012
Flight Time Uncertainty Prediction

- Flight time uncertainty
  - Strongly correlated with meteorological conditions

Calm Condition

Severe Condition

Excessive time interval

Insufficient time interval

Proper Prediction

Efficient time interval

Safe time interval

- Flight time uncertainty prediction based on meteorological conditions will enhance both safety and efficiency of time-based operations.
Studies on Trajectory Uncertainty Modeling using Meteorological Conditions

- Wind uncertainty modeling using ensemble weather forecast
  - Zheng et al., AIAA ATIO 2011, Rivas et al., SESAR 2016, etc., ICRAT 2016

- Flight Time uncertainty modeling by meteorological condition
  - Correlation between flight time uncertainty and meteorological conditions
    - Takeichi et al., DASC 2017
  - Prediction Modeling of GS fluctuation using flight speed and meteorological conditions
    - Accurate prediction of flight time uncertainty is feasible
      - Valid only for a specific flight distance
        - Takeichi et al., TRC 2018
Objective: derive a model to directly predict flight time uncertainty
- A function of meteorological conditions
- Capable of predicting for an arbitrary flight distance

Future 4D trajectory management concept
- Onboard ETA is downlinked to ATCo (Mutuel et al., ATM Sem. 2013)
- A similar procedure is considered in this study
  - ATCo predicts the uncertainty of ETA using a prediction model

Theoretical & data analysis
- Formulation of prediction model using the Law of uncertainty propagation
- Cluster & regression analysis to determine its parameters
  - Actual operational data & weather forecast data
Operational Data

- SSR Mode S Data
  - Ground stations in Tokyo & Sendai
  - March, June, September & December in 2015 & 2016
  - Included data
    - Aircraft type
    - Altitude, longitude and latitude
    - True track angle, magnetic heading
    - GS, TAS, IAS, Mach number
    - Data interval: 10s
Numerical Weather Forecast Data

- **Global Spectral Model**
  - provided by the Japan Meteorological Agency
  - Updated every 6 hours
  - Providing forecast data every 3 hours
- **Resolution**
  - 0.25deg in longitude and 0.2deg in latitude
  - 13 pressure altitude layers
    - 100-1000 hPa pressure altitudes
- **Nowcast data** are utilized in the analysis.
Aircraft in the cruise phase are usually controlled to maintain a specific Mach number, track angle and pressure altitude.

**Trajectory data: 62713 flights**
- Controlled cruise trajectories
  - Mach number within 0.02
  - Pressure altitude within 100ft
  - True track angle within 5deg
  - for distance > 100km
  - Pressure altitude above 25000ft

**Trajectory Data**
Flight Time Error Analysis

- Flight time error = Actual time – Predicted time
  - Predicted flight time \( T_{\text{pred}} = \frac{D}{V_{\text{GS}}^{\text{ini}}} \)
    - Calculated using GS at the moment of prediction
    - Simulates onboard short-term prediction
  - Actual flight time calculation
    - Integration of recorded GS \( \int_{0}^{T_{\text{act}}} V_{\text{GS,act}} dt = D \)
  - Flight time error: \( T_{\text{err}} \triangleq T_{\text{act}} - T_{\text{pred}} \)

Error distribution

- \( mean = -0.19 \text{ sec} \)
- \( STD = 7.24 \text{ sec} \)
- \( RMS = 7.24 \text{ sec} \) at 200km
Onboard flight time prediction: $t_f = \frac{D}{V_{GS, ini}}$

GS is a function of Mach number and meteorological conditions:
- Mach number, along-track wind, cross-track wind, temperature, true altitude
  
  \[ t_f = f(D, M, W_t, W_c, T, h) \]

Total differential becomes:

\[
dt_f = \frac{\partial t_f}{\partial M} dM + \frac{\partial t_f}{\partial W_t} dW_t + \frac{\partial t_f}{\partial W_c} dW_c + \frac{\partial t_f}{\partial T} dT + \frac{\partial t_f}{\partial h} dh
\]

Applying the Law of uncertainty propagation, the variance of the flight time prediction error can be obtained as:

\[
\sigma_{t_f}^2 = \left(\frac{\partial t_f}{\partial M}\right)^2 \sigma_M^2 + \left(\frac{\partial t_f}{\partial W_t}\right)^2 \sigma_{W_t}^2 + \left(\frac{\partial t_f}{\partial W_c}\right)^2 \sigma_{W_c}^2 + \left(\frac{\partial t_f}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial t_f}{\partial h}\right)^2 \sigma_h^2
\]

\[+ \left(\frac{\partial t_f}{\partial M}\right) \left(\frac{\partial t_f}{\partial W_t}\right) \sigma_{MW_t} + \cdots + \left(\frac{\partial t_f}{\partial T}\right) \left(\frac{\partial t_f}{\partial h}\right) \sigma_{Th}\]
Theoretical Basis

- Derivation of partial differential terms \( \frac{\partial t_f}{\partial M} , \frac{\partial t_f}{\partial W_t}, \frac{\partial t_f}{\partial W_c}, \frac{\partial t_f}{\partial T}, \frac{\partial t_f}{\partial h} \)

- Geometrical calculations
  - GS is obtained as a sum of along-track component of TAS and tailwind.
    \[
    V_{GS} = TAS_{tr} + W_t
    \]
  - along-track component of TAS is obtained from measured TAS and crosswind:
    - Measured TAS is expressed by Mach number.
      \[
      TAS_{tr} = \sqrt{TAS_m^2 - W_c^2} = \sqrt{M^2 \kappa RT - W_c^2} \quad \Rightarrow \quad t_f = \frac{D}{\sqrt{M^2 \kappa RT - W_c^2 + W_t}}
      \]
    - Derivation of partial differential by Mach number
      \[
      \frac{\partial t_f}{\partial M} = -\frac{D}{\left(\sqrt{M^2 \kappa RT - W_c^2 + W_t}\right)^2} \cdot \frac{M \kappa RT}{\sqrt{M^2 \kappa RT - W_c^2}} = -\frac{D}{V_{GS}} \cdot \frac{M \kappa RT}{V_{TAS_{tr}}}
      \]
Theoretical Basis

- Derivation of partial differential with respect to tailwind & crosswind

\[
\frac{\partial t_f}{\partial W_t} = \frac{D}{\left(\sqrt{M^2 \kappa RT - W_c^2} + W_t\right)^2} = -\frac{D}{V_{GS}^2} \\
\frac{\partial t_f}{\partial W_c} = \frac{D}{\left(\sqrt{M^2 \kappa RT - W_c^2} + W_t\right)^2} \cdot \frac{W_c}{\sqrt{M^2 \kappa RT - W_c^2}} = \frac{D}{V_{GS}^2} \cdot \frac{W_c}{V_{TAStr}}
\]

- Derivation of partial differential by temperature

\[
\frac{\partial t_f}{\partial T} = -\frac{D}{\left(\sqrt{M^2 \kappa RT - W_c^2} + W_t\right)^2} \cdot \frac{M^2 \kappa R}{2\sqrt{M^2 \kappa RT - W_c^2}} = -\frac{D}{V_{GS}^2} \cdot \frac{M^2 \kappa R}{2V_{TAStr}}
\]
Theoretical Basis

- \( t_f = \frac{D}{\sqrt{M^2 \kappa RT - W_c^2 + W_t}} \) does not include true altitude.

- Variation of true altitude is also a cause of GS fluctuations.

\[ \frac{\partial t_f}{\partial h} \]

is obtained from the Law of conservation of mechanical energy:

\[ \frac{1}{2} V_{GS}^2 + gh = \text{const.} \]

\[ dV_{GS} = -\frac{g}{V_{GS}} dh \]

\[ t_f = \frac{D}{V_{GS}} \]

\[ dt_f = -\frac{D}{V_{GS}^2} dV_{GS} \]

\[ dt_f = \frac{Dg}{V_{GS}^3} dh \]
Theoretical Basis

- Total differential equation is obtained according to:

\[
dt_f = -\frac{D}{V_{GS}^2}dW_t + \frac{D}{V_{GS}^2} \frac{W_c}{V_{TAStr}} dW_c - \frac{D}{V_{GS}^2} \frac{M^2\kappa R}{2V_{TAStr}} dT - \frac{D}{V_{GS}^2} \frac{M\kappa RT}{V_{TAStr}} dM + \frac{Dg}{V_{GS}^3} dh
\]

- Application of the Law of uncertainty propagation

  - Assumptions
    - Wind fluctuation is homogeneous in all directions
    - No correlation among variables except for the temperature and true altitude

\[
\sigma_{t_f}^2 = \left(\frac{D}{V_{GS}^2}\right)^2 \left(1 + \left(\frac{W_c}{V_{TAStr}}\right)^2\right) \sigma_w^2 + \left(\frac{D}{V_{GS}^2}\right)^2 \left(2V_{TAStr} M^2\kappa R\right) \sigma_T^2 + \left(\frac{D}{V_{GS}^2}\right)^2 \left(\frac{M\kappa RT}{V_{TAStr}}\right)^2 \sigma_M^2
\]

\[+ \left(\frac{D}{V_{GS}^2}\right)^2 \left(\frac{g}{V_{GS}}\right)^2 \sigma_h^2 - \frac{D^2}{V_{GS}^5} \frac{M^2\kappa R}{V_{TAStr}} g\sigma_{Th}\]

This equation outputs the ideal flight time uncertainty if the absolute weather conditions could be applied.
Data Analysis

- Numerical weather forecast are provided as discrete data
  - discrepancy in flight time uncertainty prediction is inevitable even the weather forecast is completely accurate.

- Coefficient $\alpha$ is introduced to compensate for such discrepancy.

- $\sigma_M$ is treated as a constant value.
  - unavailable in advance

- Coefficient $\alpha$ is determined through analysis of measured data.

\[
\sigma_{t_f}^2 = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5
\]

\[
x_1 \triangleq \left( \frac{D}{V^2_{GS}} \right)^2 \left( 1 + \left( \frac{W_c}{V_{TAStr}} \right)^2 \right) \sigma_w^2, 
\]

\[
x_2 \triangleq \left( \frac{D}{V^2_{GS}} \right)^2 \left( \frac{M^2 \kappa R}{2V^{2}_{TAStr}} \right)^2 \sigma_T^2,
\]

\[
x_3 \triangleq \left( \frac{D}{V^2_{GS}} \right)^2 \left( \frac{M \kappa R T}{V_{TAStr}} \right)^2, 
x_4 \triangleq \left( \frac{D}{V^2_{GS}} \right)^2 \left( \frac{g}{V_{GS}} \right)^2 \sigma_h^2, 
x_5 \triangleq - \frac{D^2}{V^5_{GS}} \frac{M^2 \kappa R}{V_{TAStr}} g \sigma_{Th}
\]
Data Analysis

- Cluster analysis & Multiple linear regression
  - to determine coefficients of linear equation of statistical values

- Cluster Analysis
  - Algorithm: Gaussian Mixture Models (GMM) using Expectation-Maximization (EM) Algorithm
    - Distribution of each parameter in each cluster be akin to Gaussian
    - Suitable to evaluate statistical values such as mean, variance and covariance

- Multiple linear regression
  - To determine coefficients of linear equation
  \[
  \sigma_{t_f}^2 = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5
  \]

\[
  x_1 \triangleq \left( \frac{D}{V_{GS}^2} \right)^2 \left( 1 + \frac{W_c}{V_{TAStr}} \right)^2 \sigma_W^2,
  x_2 \triangleq \left( \frac{D}{V_{GS}^2} \right)^2 \left( \frac{M^2 \kappa R}{2V_{TAStr}} \right)^2 \sigma_T^2,
\]

\[
  x_3 \triangleq \left( \frac{D}{V_{GS}^2} \right)^2 \left( \frac{M \kappa RT}{V_{TAStr}} \right)^2,
  x_4 \triangleq \left( \frac{D}{V_{GS}^2} \right)^2 \left( \frac{g}{V_{GS}} \right)^2 \sigma_h^2,
  x_5 \triangleq - \frac{D^2}{V_{GS}^5} \frac{M^2 \kappa R}{V_{TAStr}} \sigma_{Th}
\]
Data Analysis

- Onboard prediction uses the GS at the moment of prediction
- ATCo predicts the flight time uncertainty based on,
  - Mach number
  - Weather forecast from the time of calculating ETA to the ETA
- Flight time is predicted from the initial values of each data sample.
  - Variances are calculated as mean squares of difference from initial values to simulate the behavior of deviation from the initial value
    - instead of the difference from Means
    \[
    \sigma^2_W = \frac{1}{N} \sum_{i=1}^{N} \| W_i - W_{ini} \|^2 \left( W = (W_t, W_c)^T \right), \sigma^2_T = \frac{1}{N} \sum_{i=1}^{N} (T_i - T_{ini})^2,
    \]
    \[
    \sigma^2_h = \frac{1}{N} \sum_{i=1}^{N} (h_i - h_{ini})^2, \sigma_{Th} = \frac{1}{N} \sum_{i=1}^{N} (T_i - T_{ini}) (h_i - h_{ini})
    \]
- Data splitting
  - 50% samples for modeling
  - 50% samples for evaluation
Cluster Analysis

- **GMM-EM clustering**
  - Feature parameters: \((x_{1,\text{ini}}, x_{2,\text{ini}}, x_{3,\text{ini}}, x_{4,\text{ini}}, x_{5,\text{ini}})\)
  - Number of clusters: 120
    - determined according to BIC analysis
  - Samples outside 4 STD range are eliminated

- **Results**
  - 58 clusters including more than 50 samples
  - Following parameters are calculated for each cluster
    - Variance of flight time prediction error \(\sigma_{t_f,act}^2 = \text{mean}(t_{err}^2)\)
    - Mean of parameters \(x_1 \sim x_5\)
Multiple Linear Regression Analysis

- Correlation between $\sigma_{t_f,act}^2$ and $x_1 \sim x_5$
- All parameters have significant correlation
- used in multiple linear regression

<table>
<thead>
<tr>
<th>Correlation Coefficient</th>
<th>P Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0.98</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.76</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.96</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.82</td>
</tr>
<tr>
<td>$x_5$</td>
<td>0.61</td>
</tr>
</tbody>
</table>

\[
x_1 \triangleq \left( \frac{D}{V_{GS}^2} \right)^2 \left( 1 + \left( \frac{W_c}{V_{TAStr}} \right)^2 \right) \sigma_{W}^2, \quad x_2 \triangleq \left( \frac{D}{V_{GS}^2} \right)^2 \left( \frac{M^2 \kappa R}{2V_{TAStr}} \right)^2 \sigma_{T}^2, \]
\[
x_3 \triangleq \left( \frac{D}{V_{GS}^2} \right)^2 \left( \frac{M \kappa RT}{V_{TAStr}} \right)^2, \quad x_4 \triangleq \left( \frac{D}{V_{GS}^2} \right)^2 \left( \frac{g}{V_{GS}} \right)^2 \sigma_{h}^2, \quad x_5 \triangleq -\frac{D^2}{V_{GS}^5} \frac{M^2 \kappa R}{V_{TAStr}} g \sigma_{Th}
\]
Multiple Linear Regression Analysis

- **Regression function**

\[
\sigma_{t_f,est}^2 = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5
\]

\[
= 0.28x_1 + 11.40x_2 + 5.6 \times 10^{-5} x_3 + 0.40x_4 - 12.7 x_5
\]

<table>
<thead>
<tr>
<th>Error RMS [s^2]</th>
<th>R^2</th>
<th>P Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.3</td>
<td>1.00</td>
<td>&lt; 0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Error</th>
<th>T Value</th>
<th>P Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_1</td>
<td>$1.12 \times 10^{-2}$</td>
<td>24.5</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>x_2</td>
<td>$6.16 \times 10^{-1}$</td>
<td>18.5</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>x_3</td>
<td>$4.94 \times 10^{-6}$</td>
<td>11.3</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>x_4</td>
<td>$1.14 \times 10^{-1}$</td>
<td>3.55</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>x_5</td>
<td>$1.05 \times 10^{0}$</td>
<td>-12.1</td>
<td>&lt; 0.01</td>
</tr>
</tbody>
</table>
Evaluation of Direct Prediction

- Direct prediction model is expected to accurately predict the flight time uncertainty
  - non-linear increase with distance
  - for an arbitrary weather condition

- Focus of evaluation to clarify the advantage of the direct prediction model

- Reference: conventional static prediction model
  \[
  \sigma_{t_f, st}^2 = \left( \frac{D}{V_{GS}^2} \right)^2 \sigma_{GS, st}^2
  \]

- \( \sigma_{GS, st}^2 \) is evaluated as 8.62\( [m^2/s^2] \)
Evaluation on Uncertainty Increase

- RMS values of actual arrival time error, uncertainty estimated from both direct & static prediction models from 100km to 500km
  - Direct prediction model is able to predict increase behavior.
  - Conventional static prediction model is unable to predict it.
  - Linear proportional to flight distance

![Graphs showing actual and predicted RMS values for distances 200km and 500km.](image-url)
Evaluation in Severe/Calm Conditions

- Normalized flight time error
  - Actual flight time error/predicted RMS value
- Accurate prediction: RMS of normalized error = 1
  - RMS > 1 / RMS < 1 means underestimation/overestimation
- 1/4th of trajectories each from largest and smallest $\sigma_{tf,est}^2$
- To clarify advantage of the proposed model in severe and calm conditions

\[
V_{GS} \rightarrow D \rightarrow RMS
\]

- RMS = 1 $\rightarrow$ Accurate Prediction
- RMS > 1 $\rightarrow$ Underestimation
- RMS < 1 $\rightarrow$ Overestimation
Evaluation in Severe/Calm Conditions

- Evaluation result
  - RMSs by conventional static prediction model: 1.3 times larger/smaller at 200km
    - overestimation/underestimation
  - RMSs by direct prediction model: closer to 1.0 in all cases

- Distributions of normalized flight time prediction error at 200km
  - Direct prediction: close to standard normal distribution in any conditions
  - Conventional static prediction lacks accuracy in irregular conditions

![Graph showing comparison between conventional and direct models]

- RMS of Normalized Flight Time Error
  - Large, direct
  - Large, conventional
  - Small, direct
  - Small, conventional

- Distributions of Normalized Flight Time Error
  - N=4902 at 200km
  - Direct, conventional, N(0,1)
Conclusion & Future Works

- **Conclusion**
  - Direct prediction model of flight time uncertainty
    - As a function of flight condition and weather forecast
    - Capable of predicting at an arbitrary flight distance
    - Coefficients were determined by cluster and regression analysis
  
  - Obtained results show that the proposed model is capable of accurately predicting
    - Non-linear increase of flight time uncertainty
    - Flight time uncertainty at arbitrary weather conditions
  
  - Proposed direct prediction model facilitates safety and efficiency simultaneously in future time-based operations.

- **Future works**
  - Improving the functionality of the prediction model
    - Capable of uncertainty prediction in descent & climb phases
    - A significant contribution towards future time-based operations