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A GEOMETRICAL APPROACH TO CONFLICT PROBABILITY ESTIMATION

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1. Abstract

A method of estimating conflict probability is described. Initially the method is presented considering only along-track errors but subsequently cross-track errors are included. Graphs of conflict probability illustrate limits on the levels of performance that may be attained in medium-term conflict detection and resolution (in the absence of measures to reduce along-track errors). It is shown that, under certain assumptions, the minimum displacement of one aircraft from the other has a normal distribution, and that the minimum distance between them has a folded normal distribution.

2. Introduction

Conflict prediction is based on trajectory prediction, which suffers from various sources of error. These errors may be due to imperfect modelling of aircraft behaviour, due to imperfect knowledge of parameters, or due to unpredicted variations in the speed and direction of the wind. This last source of error will be of particular interest in this paper, the other types of error being, in principle, remediable. As a consequence of these errors conflicts which are predicted at a certain time may not occur, and conflicts may occur which were not predicted at that time. Methods have been developed to estimate the likelihood or probability that encounters will develop into conflicts. These have theoretical value as they indicate limits on the levels of performance that may be attained in conflict detection and resolution in the absence of further measures to control errors. They may also be useful in the implementation of conflict detection and resolution applications, for example, to determine when to notify controllers of a possible conflict.

A review of prior work on conflict probability estimation methods is included in [6]. Similar approaches to conflict probability estimation are described in [1, 2]. The along-track and cross-track position errors of both aircraft involved in an encounter are combined into an elliptical error distribution centred on one of the aircraft (the "intruder" or "stochastic" aircraft). A disc representing the required separation is centred on the other aircraft (the "reference" aircraft). The aircraft move one relative to the other. The probability of conflict corresponds to that part of the elliptical error distribution that at some time overlaps the separation disc. In [2] the elliptical error distribution is transformed into a circular one, and the separation disc is transformed into an elliptical conflict zone. The movement of this zone relative to the stochastic aircraft sweeps out a rectangular region termed the extended conflict zone. The probability of conflict corresponds to that part of the circular error distribution that overlaps the extended conflict zone. It is assumed that the planned velocities and prediction errors are constant in a region of possible conflict. This method is used in CTAS (Centre/Tracon Automation System) to determine when to notify controllers of a possible conflict. An empirical test of the method is described in [5].

In [8] the geometry of the encounter defines an elliptical conflict region in a rectangular co-ordinate system in which the axes are the along-track distances flown by the aircraft The probability of conflict corresponds to a slice of the distribution of the ratio of the average speeds of the aircraft. However, the method described in [8] depends upon a binomial expansion that is only valid for a limited range of error models. [8] does not allow for cross-track errors, arguing that these are small compared with along-track errors for FMS controlled aircraft. [8] also describes how conflict probability can be estimated if aircraft speeds change prior to entering a region of possible conflict or if the aircraft turn within that region.

This paper presents a slightly different method from that described in [8] which does not rely upon a binomial expansion and which is therefore not subject to the limited range of error model. Initially the method is presented considering along-track errors but subsequently cross-track errors are included. Further, it is shown that, under certain assumptions, the minimum displacement of one aircraft from the other has a normal distribution, and the minimum distance has a "folded" normal distribution.

Only horizontal conflict probability estimation is described. In practice this would need to be complemented by a method for estimating vertical conflict probability.

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3. Calculation of conflict probability

3.1 Encounter geometry

Consider an encounter between two aircraft X and Y in the horizontal plane. Suppose that that the tracks are straight in the region where a conflict may occur (although they may contain turns before and after this region).

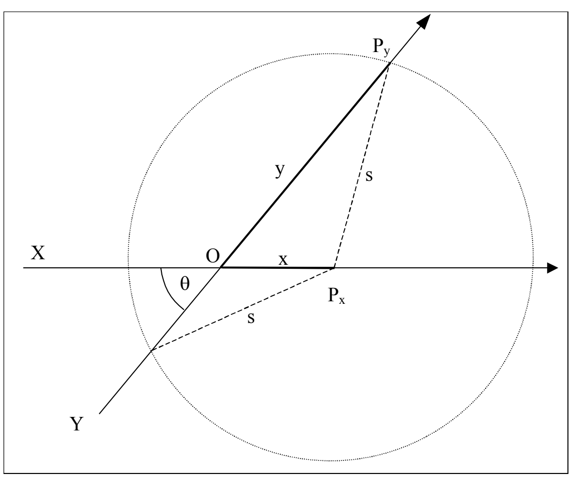


Figure 1

Let

- O be the crossing point
- θ be the crossing angle
- x be the along track distance flown by aircraft X at time t, with the crossing point O as the origin
- P_x be the position of aircraft X
- y be the along track distance flown by aircraft Y at time t, with the crossing point O as the origin
- P_y be the position of aircraft Y when it is just separated from aircraft X
- s be the separation required between the two aircraft

The aircraft will be in conflict if aircraft Y lies within a circle of radius s centred on P_x. If the aircraft are just separated aircraft Y would lie on the circle. Applying the cosine rule to triangle OP_xP_y

$$x^2 + y^2 - 2xy\cos\theta = s^2 \qquad (1)$$

This equation describes an ellipse and is plotted below for a range of crossing angles between 0 and 180 degrees with a required separation of 5 nautical miles.

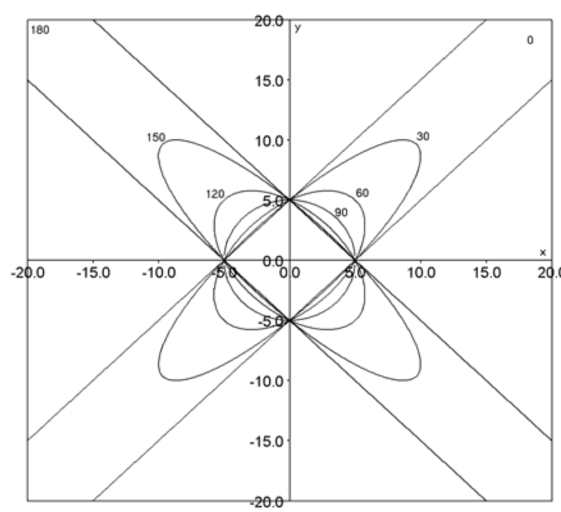


Figure 2

The along-track distances of both aircraft from the crossing point at any time together define a point in this diagram. For example, assuming that the aircraft will pass through the crossing point at similar times, then at some time before the encounter their along-track distances will define a point (x_0, y_0) in the lower left-hand quadrant. At a time after the encounter their distances will define a point in the upper right-hand quadrant. The locus of the point through time defines a path in this diagram. If the path passes around the ellipse then the aircraft will not conflict, and vice-versa.

The gradient of the path at any point is the ratio of the speeds of the aircraft at that point, since

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{v_Y}{v_X} \qquad (2)$$

Consider the case that the ground speeds of the aircraft are constant throughout the encounter. In this case the path is a straight line whose gradient m is the ratio of the speeds.

$$m = \frac{v_{PY}}{v_{PX}} \qquad (3)$$

Whether or not a conflict will occur, depends upon whether the line passes through the ellipse, as illustrated below:

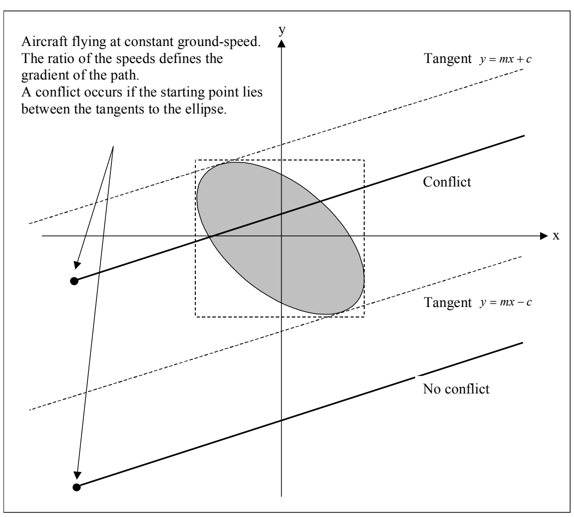


Figure 3

It can be seen that the initial along-track distances will give rise to a conflict if they define a starting point that lies between the tangents to the ellipse.

The tangents have equations:

$$y = mx \pm c \qquad (4)$$

where c can be found by substituting $y = mx + c$ into the equation of the ellipse, and requiring that there be a single solution:

$$c = s/\lambda \qquad (5)$$

where $\lambda \equiv \dfrac{\sin\theta}{\sqrt{m^2 - 2m\cos\theta + 1}}$

A given point (x', y') lies between the two tangents if

$$-c < y' - mx' < c \qquad (6)$$

that is, if the intercept of the line on the y-axis lies between the intercepts of the two tangents. Using (5) this condition can be rewritten:

$$-s < \lambda(y' - mx') < s \qquad (7)$$

which suggests that the term between the inequalities represents the magnitude of the displacement (or relative position vector) between the aircraft when they are at minimum distance, hereafter referred to as the "minimum displacement".[1]

$$d_{\min} = \lambda(y' - mx') \qquad (8)$$

The minimum displacement lies along a vector that is perpendicular to the relative velocity. Note that it is a simple linear combination of the initial along-track distances. The minimum distance is the absolute value of the minimum displacement.

[1] That this is the case can be demonstrated by finding $d_{\min}$ such that the line through (x', y') with gradient m is tangent to $x^2 + y^2 - 2xy\cos\theta = d_{\min}^2$, i.e. there is one and only one point of intersection of the line with the ellipse, or, directly by calculation of the magnitude of the relative position vector when it is perpendicular to the relative velocity.

Consider λ as a function of crossing angle θ. In the case that the speeds of the aircraft are identical, i.e. $m = 1$, then $\lambda = \cos(\theta/2)$. However, this is a very special case. In general the speeds of the aircraft are not identical, i.e., $m \neq 1$, and the behaviour of λ for small angles is quite different. In this case, for small θ, $\lambda \approx \theta/|1-m|$, As $\theta \to 0$, $\theta/|1-m| \to 0$, whereas $\cos(\theta/2) \to 1$. λ is plotted as a function of θ for m = 0.8, 0.9, 1.0, 1.1, and 1.2 in figure 4.

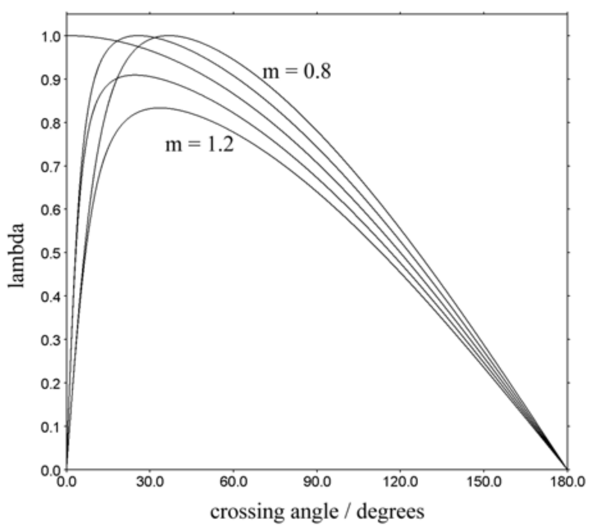


Figure 4

3.2 Along-track errors

To begin with only along-track errors are considered. Cross-track errors will be included in the next section. The effect of an along-track error is to advance or retard an aircraft along its track. The along-track error is acquired as the aircraft progresses along its track, so that in reality its speed will differ from the predicted speed and is not constant. It is assumed, however, that within the range of along-track distances for which conflict is possible, the along-track error in an aircraft's position is approximately constant and that the aircraft flies with its predicted speed. This assumption is also made in [2].

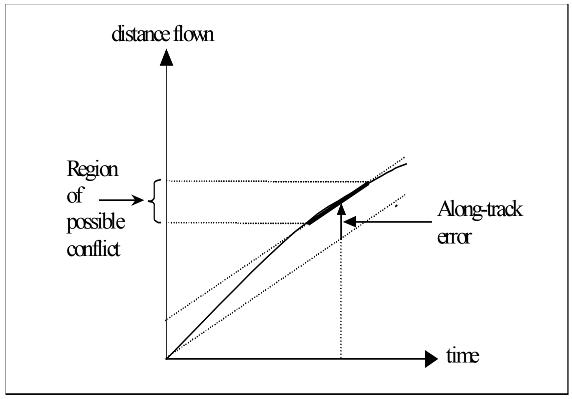


Figure 5

Having made this assumption, we can consider an alternative process by which an aircraft might acquire a given along-track error, namely, that the error is present in the initial along-track distance of the aircraft and that it then flies with constant speed. Subject to this assumption, an observer watching the movement of an aircraft within the region of possible conflict cannot distinguish which process gave rise to the along-track error.

Let the error in the predicted position of aircraft X along its track at a time τ into the future be a random variable denoted by $\alpha_X(\tau)$. Similarly let the error in the predicted position of aircraft Y along its track be a random variable denoted by $\alpha_Y(\tau)$ Subject to the above assumption, these errors can be considered as being equivalent to errors in the initial along-track distances:

$$x' = x_0 + \alpha_X(\tau)$$
$$y' = y_0 + \alpha_Y(\tau) \qquad (9)$$

Since the along-track errors $\alpha_X(\tau)$ and $\alpha_Y(\tau)$ are random variables so too are x' and y'.
Let $i \equiv y' - mx'$ be the intercept on the y-axis of a line with gradient m through (x', y'). Since x' and y' are random variables so too is the intercept.

$$i = (y_0 - mx_0) + \alpha_Y(\tau) - m\alpha_X(\tau) \qquad (10)$$

According to [2, 5], the along-track distance error at a time for aircraft in level flight is well modelled by a normal distribution. This statement effectively summarises the cumulative effect over time of an underlying stochastic process, without saying anything about its time-varying characteristics. We can write

$$\alpha(\tau) = a(\tau) A_\tau \qquad (11)$$

where A_τ is a normally distributed random variable with zero mean and unit variance and $a(\tau)$ models the growth of the standard deviation of the along-track error with time. According to [2, 5], for prediction intervals of up to 20 minutes, the standard deviation of the along-track distance error grows linearly with time, i.e., $a(\tau) = a\tau$ where a is the rate of growth of the standard deviation of the along-track error.

Substituting (11) into (10)

$$i = (y_0 - mx_0) + a(\tau)A_Y - ma(\tau)A_X \qquad (12)$$

The along-track errors are normally distributed. If we make the assumption that the along-track errors are independent, then the intercept i is the sum of a constant and two independent, normally distributed random variables. The sum of independent, normally distributed random variables is also normally distributed, with a mean equal to the sum of the means of the individual distributions, and variance equal to the sum of the variances of the individual distributions. Consequently i is normally distributed with mean μ_i and variance $\sigma_i^{\,2}$ given by:

$$\mu_i = (y_0 - mx_0)$$

$$\sigma_i^{\,2} = a(\tau)^2 (1 + m^2) \qquad (13)$$

The probability of conflict corresponds to the part of this distribution that lies between $-c$ and $+c$, i.e.,

$$P_{conflict} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-c}^{c} \exp\left(-\frac{1}{2}\left(\frac{i-\mu_i}{\sigma_i}\right)^2\right) di$$

$$= \frac{1}{\sqrt{2\pi}} \int_{(-c-\mu_i)/\sigma_i}^{(c-\mu_i)/\sigma_i} \exp\left(-z^2/2\right) dz \qquad (14)$$

or

$$P_{conflict} = C\left(\frac{c-\mu_i}{\sigma_i}\right) - C\left(\frac{-c-\mu_i}{\sigma_i}\right) \qquad (15)$$

where $C(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-z^2/2\right) dz$

$C(x)$ is the cumulative probability[1] or distribution function of a normally distributed random variable with a mean of zero and variance of one (and can be evaluated using a pre-computed table and interpolation).

Alternatively, since the minimum displacement $d_{\min}$ differs from the intercept $y' - mx'$ only by a scaling factor λ (see equation (8)) it also has a normal distribution with parameters

$$\mu_d = \lambda\mu_i = \lambda(y_0 - mx_0)$$

$$\sigma_d^{\,2} = \lambda^2\sigma_i^{\,2} = \lambda^2 a(\tau)^2 (1 + m^2)$$

(16)

In this case the mean is the nominal minimum displacement in the absence of errors.

The probability of conflict is then the probability that the minimum displacement lies between $-s$ and s, that is,

[1] The cumulative probability function $\Phi(x)$ of a random variable X is the probability that X takes a value less than or equal to x, i.e., $\Phi(x) = P(X \le x)$. For a continuous random variable the cumulative probability function $\Phi(x)$ is the area beneath the probability density function $\phi(x)$ over the interval $(-\infty, x)$.

$$P_{conflict} = C\left(\frac{s-\mu_d}{\sigma_d}\right) - C\left(\frac{-s-\mu_d}{\sigma_d}\right) \quad (17)$$

Since the standard deviation of the along-track error is a function of time, it is necessary to choose a time for which it is evaluated. A simple approximation is to use the predicted (or nominal) time to minimum distance. However, this is not appropriate when aircraft are potentially in conflict over a long period, which is the case when the speeds are similar and the angle of approach is small. Suppose that the predicted minimum distance is less than the required separation. If the along-track errors were evaluated at the predicted time to minimum distance, the resulting estimate of the probability of conflict would be an underestimate. It is preferable to err on the side of overestimating the probability of conflict, consequently, in cases where a loss of separation is predicted the along-track errors are evaluated at the predicted time of first loss of separation.

The probability that an encounter will be conflict-free is

$$P_{conflict-free} = 1 - P_{conflict} \quad (18)$$

The assumption that the along-track errors of the aircraft are independent may well be poor for encounters in which aircraft are close to one another for a long period. This is the case for encounters in which the aircraft have similar speeds and the angle of approach is small. In such cases the along-track errors due to wind are more likely to be positively correlated, which reduces uncertainty. For predicted minimum distances less than the required separation the estimate of probability of conflict given here (15 and 17) will be an underestimate, and similarly for predicted minimum distances greater than the required separation the probability that the encounter will be conflict-free (18) will be an underestimate. For the purposes of illustration consider the extreme case in which the speeds of both aircraft are the same and the along-track errors of both aircraft are identical at all times. In this case, in equation (10) the two random terms cancel and the prediction of a conflict or non-conflict then depends only upon whether the predicted minimum distance is less than or greater than the required separation.

3.3 Cross-track errors

The analysis so far has only taken into account along-track errors. A major difference between the two types of error is that flight management systems limit the magnitude of cross-track errors. Depending upon the error model, cross-track errors may have a significant impact on the probability of conflict for marginal conflicts (predicted minimum distance close to the required separation) in which the crossing angle is near to 180 degrees. It may also be convenient to decompose other kinds of error into along-track and cross-track components.

The effect of cross-track errors is to move the crossing point of the tracks. The crossing angle remains unchanged. An observer who moves with the crossing point sees cross-track errors as errors in along-track distance from the crossing point.

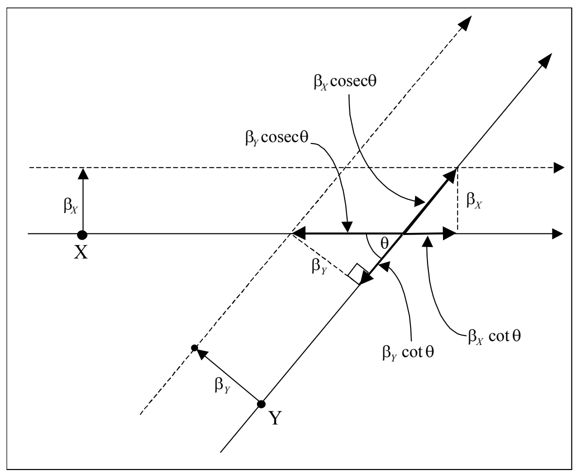


Figure 6

If both aircraft are moving towards the crossing point, a cross-track error β_Y in the position of aircraft Y with respect to the ground (see figure 6) reduces the along-track distance of aircraft Y from the crossing point by $\beta_Y \cot\theta$ and reduces the along-track distance of aircraft X from the crossing point by $\beta_Y \operatorname{cosec}\theta$. Similarly a cross-track error β_X in the position of aircraft X with respect to the ground increases the along-track distance of aircraft X from the crossing point by $\beta_X \cot\theta$ and increases the along-track distance of aircraft Y from the crossing point by $\beta_X \operatorname{cosec}\theta$.

Consequently, we can consider cross-track errors β_X and β_Y as being equivalent to along-track errors α'_X and α'_Y where

$$\begin{aligned} \alpha'_X &= -\beta_X \cot\theta + \beta_Y \operatorname{cosec}\theta \\ \alpha'_Y &= +\beta_Y \cot\theta - \beta_X \operatorname{cosec}\theta \end{aligned} \quad (19)$$

The combined along-track errors with respect to the crossing point are then:

$$\begin{aligned} \gamma_x(\tau) &= \alpha_X(\tau) - \beta_X \cot\theta + \beta_Y \operatorname{cosec}\theta \\ \gamma_y(\tau) &= \alpha_Y(\tau) + \beta_Y \cot\theta - \beta_X \operatorname{cosec}\theta \end{aligned} \quad (20)$$

where $\alpha(\tau)$ is the along-track error with respect to the ground.

Assuming again that these errors are approximately constant in the region of possible conflict, then, as described earlier, they may be considered as being equivalent to errors in the initial along-track distances of the aircraft, i.e.,

$$\begin{aligned} x' &= x_0 + \alpha_X(\tau) - \beta_X \cot\theta + \beta_Y \operatorname{cosec}\theta \\ y' &= y_0 + \alpha_Y(\tau) + \beta_Y \cot\theta - \beta_X \operatorname{cosec}\theta \end{aligned} \quad (21)$$

As before, the intercept is given by $i \equiv y' - mx'$. Then, regrouping terms in the same random variable,

$$\begin{aligned} i = & \left(y_0 - mx_0\right) \\ & + \alpha_Y(\tau) - m\alpha_X(\tau) \\ & + \beta_X\left(m\cot\theta - \cos ec\theta\right) \\ & + \beta_Y\left(\cot\theta - m\cos ec\theta\right) \end{aligned} \quad (22)$$

According to [2], the cross-track error can be modelled by a normal distribution with a fixed variance, i.e.,

$$\beta = bB \quad (23)$$

where B is a normally distributed random variable with zero mean and unit variance, and b is the (constant) standard deviation of the cross-track error. However, this model of cross-track error is questionable for small prediction times: it seems preferable to suppose that initially cross-track error grows with time, but is limited in magnitude. Slightly more generally we will write:

$$\beta = b(\tau)B \quad (24)$$

where $b(\tau)$ represents a model of how the standard deviation of the cross-track error grows with time.

A simple model of how the standard deviation of the cross-track error might grow is given by

$$b(\tau) = b\left(1 - \exp\left(-\tau/\tau_c\right)\right)$$

where τ_c is a time constant.

Setting b = 1 nm and τ_c = 4 minutes gives an initial rate of growth of cross-track error of 0.25 nm/minute, which is the same as for along-track error. This model and these values are illustrative only and are not based on an analysis of real data.

Substituting the normally distributed models for along-track error (10) and cross-track error (24) into (22)

$$\begin{aligned} i = & \left(y_0 - mx_0\right) \\ & + a(\tau)A_Y - ma(\tau)A_X \\ & + b(\tau)B_X\left(m\cot\theta - \operatorname{cosec}\theta\right) \\ & + b(\tau)B_Y\left(\cot\theta - m\operatorname{cosec}\theta\right) \end{aligned} \quad (25)$$

This is a sum of normally distributed random variables. Assuming that these variables are independent, then their sum is also normally distributed with mean μ_i and variance σ_i^2 given by:

$$\begin{aligned} \mu_i &= \left(y_0 - mx_0\right) \\ \sigma_i^2 &= a(\tau)^2\left(1 + m^2\right) \\ & + b(\tau)^2\left(m\cot\theta - \operatorname{cosec}\theta\right)^2 \\ & + b(\tau)^2\left(\cot\theta - m\operatorname{cosec}\theta\right)^2 \end{aligned} \quad (26)$$

The probability of conflict is then given by:

$$P_{conflict} = C\left(\frac{c-\mu_i}{\sigma_i}\right) - C\left(\frac{-c-\mu_i}{\sigma_i}\right) \qquad (27)$$

where $C(x)$ is the cumulative probability or distribution function for a normally distributed random variable with a mean of zero and variance of one (see equation (15)).

Alternatively, since the minimum displacement $d_{\min}$ differs from the intercept $y' - mx'$ only by a scaling factor λ (see equation (8)) it also has a normal distribution with parameters

$$\mu_d = \lambda\mu_i = \lambda(y_0 - mx_0)$$

$$\begin{aligned}\sigma_d^2 &= \lambda^2\sigma_i^2 \\ &= a(\tau)^2 \frac{(1+m^2)\sin^2\theta}{m^2 - 2m\cos\theta + 1} \\ &+ b(\tau)^2 \frac{(m\cos\theta - 1)^2 + (\cos\theta - m)^2}{m^2 - 2m\cos\theta + 1}\end{aligned} \qquad (28)$$

The mean is the nominal minimum displacement in the absence of errors.

The probability of conflict is the probability that the minimum displacement lies between $-s$ and s, that is,

$$P_{conflict} = C\left(\frac{s-\mu_d}{\sigma_d}\right) - C\left(\frac{-s-\mu_d}{\sigma_d}\right) \qquad (29)$$

If the speeds of the aircraft are the same, i.e., $m = 1$, the expression for σ_d^2 simplifies to

$$\sigma_d^2 = 2a(\tau)^2\cos^2(\theta/2) + 2b(\tau)^2\sin^2(\theta/2) \qquad (30)$$

This expression shows the contribution of along-track and cross-track errors to the variance of the minimum displacement as a function of crossing angle, in this special case.

4. Graphs of conflict probability

Encounters, in which the planned velocities of the aircraft are constant, can be constructed, without taking errors into account, by specifying values of the following parameters:

- Time taken to reach minimum distance
- Minimum distance between the aircraft
- Crossing angle
- Speed of each aircraft

These parameters are sufficient to determine the initial positions and velocities of the aircraft. For an encounter so constructed, the probability that the encounter will develop into a conflict can then be calculated taking errors into account.

By constructing a range of encounters the probability of conflict, as given by equation (17) or (29), can be plotted as a function of one or more of the encounter parameters. The time taken to reach minimum distance is referred to as the "predicted" time to minimum distance, since this would indeed be a prediction if it were based on measurements of the initial positions and velocities of the aircraft. Similarly, the minimum distance used to construct the encounter is referred to as the "predicted" minimum distance.

To facilitate comparison, error models and parameters similar to those given in [2] are used. The along-track error model is that mentioned immediately after equation (11) with a rate of growth of standard deviation of 0.25 nautical miles per minute or 15 knots. In a later analysis of level flights [5] a rate of growth of along-track r.m.s. error of 0.22 nautical miles per minute is reported. The cross-track error model used is that given following equation (24), rather than the constant error model given in [2].

In all cases the speeds of both aircraft are 480 knots. The required separation is in all cases 5 nautical miles.

4.1 Probability of conflict as a function of predicted time to minimum distance

The following graph shows conflict probability against predicted time to minimum distance for selected predicted minimum distances. The crossing angle is 90 degrees in all cases (this angle is chosen simply because it is half-way between the extreme cases of identical tracks and head-on tracks).

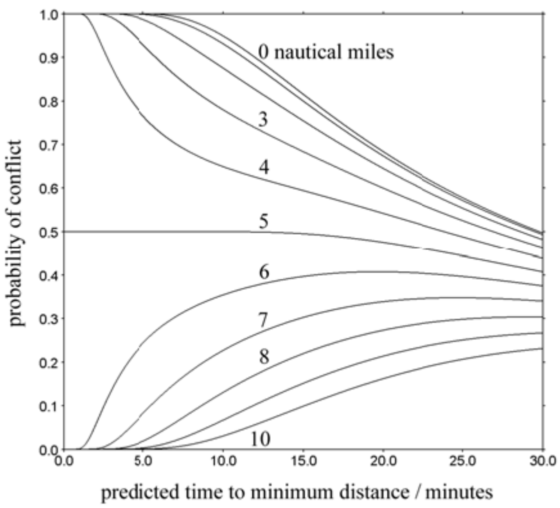


Figure 7

4.1.1 Implications for conflict detection with a fixed false alert rate

Consider a conflict detector that gives conflict indications when the probability of conflict reaches a given level. In this case the time at which an indication is given will be variable.

For example, suppose we require that only 20% of conflict indications turn out to be false. That is, we require that the probability of conflict to rise to 0.8 before indicating a possible conflict.

For a 90-degree crossing encounter in which the predicted minimum distance is 0 nautical miles, the graph shows that we will have to wait until 16 minutes to minimum distance is predicted before giving the indication.

For an encounter in which the predicted minimum distance is 2 nautical miles, we will have to wait until 13 minutes to minimum distance is predicted before giving the indication.

Finally, for an encounter in which the predicted minimum distance is 4 nautical miles, we will have to wait until 4 minutes to minimum distance is predicted before giving the indication.

4.1.2 Implications for conflict detection with a fixed warning time

Now consider a conflict detector that gives indications of possible conflicts when the predicted time to minimum distance reaches a fixed value. In this case the probability of conflict when the indication is given will be variable.

For example, suppose that conflict indications are given 20 minutes ahead of minimum distance.

For a 90-degree crossing encounter in which the predicted minimum distance is 0 nautical miles, the probability of conflict is just under 0.7.

For an encounter in which the predicted minimum distance is 2 nautical miles, the probability of conflict is about 0.65.

Finally, for an encounter in which the predicted minimum distance is 4 nautical miles, the probability of conflict is about 0.55.

It can be concluded that, with the given rate of growth along-track distance errors, for encounters of this type, if conflict indications were to be given 20 minutes before minimum distance is reached, between about a third and a half of them would turn out to be false. False alerts are undesirable because if acted upon they create unnecessary work and because the resulting resolutions may further constrain the space in which a controller must carry out future resolutions.

Now consider missed alert rates at 20 minutes to minimum distance.

For an encounter in which the predicted minimum distance is 10 nautical miles, the probability of conflict is nonetheless about 0.17, so that in such cases, about 17% of conflicts would be missed at 20 minutes.

For an encounter in which the predicted minimum distance is 8 nautical miles, the probability of conflict is about 0.27, so that in such cases, about 27% of conflicts would be missed at 20 minutes.

Finally, for an encounter in which the predicted minimum distance is 6 nautical miles, the probability of conflict is about 0.42, so that in such cases, about 42% of conflicts would be missed at 20 minutes.

To achieve lower false alert and miss rates at the same time to minimum distance would require measures to reduce along-track errors. This might be done through better wind prediction techniques, through the use of 4D flight management systems that would control along-track errors, or a combination of both.

An analysis of false and missed alert rates using real data for level flights is presented in [5].

4.1.3 Implications for conflict resolution

These last cases can also be used to illustrate the effectiveness of lateral conflict resolution.

If, following lateral conflict resolution 20 minutes before minimum distance is reached, the track crossing angle is 90 degrees, and the predicted minimum distance is 10 nautical miles, the probability of conflict is about 0.17, so that in such cases, there is a 17% chance that further intervention will be needed.

If, following lateral conflict resolution 20 minutes before minimum distance is reached, the track crossing angle is 90 degrees, and the predicted minimum distance is 8 nautical miles, the probability of conflict is about 0.27, so that in such cases, there is a 27% chance that further intervention will be needed.

If, following lateral conflict resolution 20 minutes before minimum distance is reached, the track crossing angle is 90 degrees, and the predicted minimum distance is 6 nautical miles, the probability of conflict is about 0.42, so that in such cases, there is a 42% chance that further intervention will be needed.

These examples suggest that if lateral resolution is used at times of the order of 20 minutes to minimum distance, (predicted) miss distances must be increased in order to reduce the likelihood of further intervention. Alternatively, it seems that vertical resolution is preferable in the medium-term.

This analysis supposes that the deterministic element of trajectory prediction is error-free (both horizontally and vertically). This is more likely to be reasonable for aircraft in level flight than for aircraft that are climbing or descending.

4.2 Probability of conflict as a function of crossing angle

The following graph shows probability of conflict against angle of approach for selected (predicted) times to minimum distance. In all cases the predicted minimum distance is 0 nautical miles (a conflict is predicted) and the speeds of the aircraft are identical.

Beyond about 30 degrees probability of conflict increases with crossing angle. Consequently, at a given time to minimum distance near head-on conflicts can be predicted with greater confidence than 90 degree or 45 degree conflicts. Alternatively, for a given probability of conflict (or false alert rate), head-on conflicts can be predicted earlier than 90 degree or 45 degree conflicts. As the crossing angle decreases from about 30 degrees towards zero the probability of conflict returns towards 1.0. In the case that the speeds of the aircraft are similar, this is because the time at which the error components are evaluated is the time of first loss of separation (see earlier) which in these cases may be very much earlier than the nominal time at which minimum distance will be reached. Consequently, the variance of the minimum displacement is small. For very small crossing angles the initial positions of the aircraft are such that they are already in conflict. More generally, when the speeds are dissimilar, the probability of conflict also returns towards 1.0. In these cases the scaling factor λ, which appears in the expression for the variance of the minimum displacement, tends to zero as the crossing angle tends to zero (see figure 4).

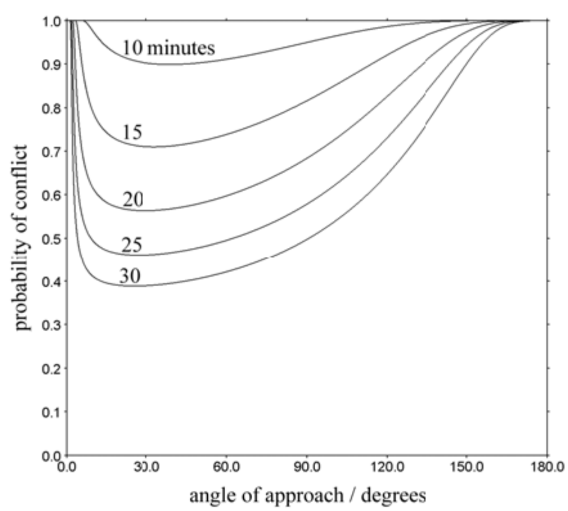


Figure 8

4.3 Impact of cross-track error model

For selected predicted minimum distances the probability of conflict is shown with and without cross-track error (figure 9). The effect of cross-track error is to lower the probability of conflict in cases where conflict is predicted (i.e. when the predicted minimum distance is less than the required separation) and to raise the probability of conflict in cases where no conflict is predicted (i.e. when the predicted minimum distance is greater than the required separation). The cross-track error model used here is that given following equation (24).[1]

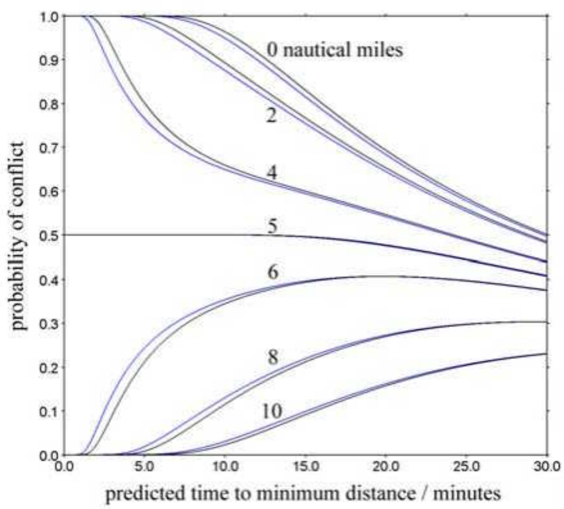


Figure 9

For 90 degree crossing encounters this model has little impact on the probability of conflict.

In the following graph (figure 10) probability of conflict is plotted against approach angle for a range of values of the predicted minimum distance at a given predicted time to minimum distance (20 minutes). It is apparent that this model does have a significant impact on probability of conflict in cases where the nominal minimum distance is close to the required separation and the angle of approach is close to 180 degrees.

[1] If this document is being viewed in colour, the black lines are the probability of conflict with no cross-track error, and the blue lines are the probability of conflict with cross-track error.

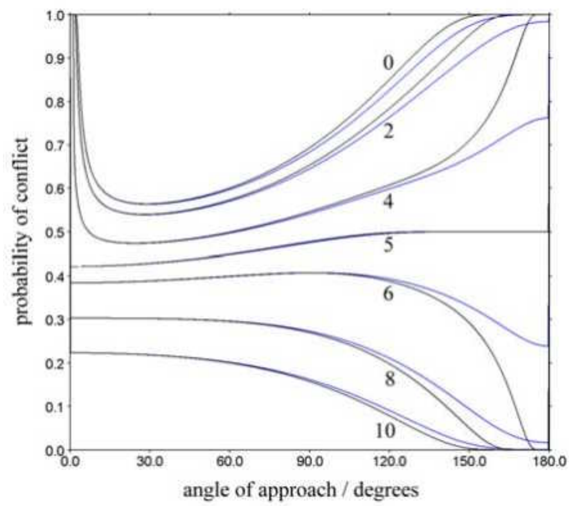


Figure 10

5. Distribution of minimum distance

The minimum distance between the aircraft is the absolute value of the minimum displacement. The distribution of the minimum distance can be obtained from the distribution of the minimum displacement by "folding" the part of the probability density function (p.d.f.) defined for negative values into the positive half-plane and adding it to the part of the p.d.f. defined for positive values.

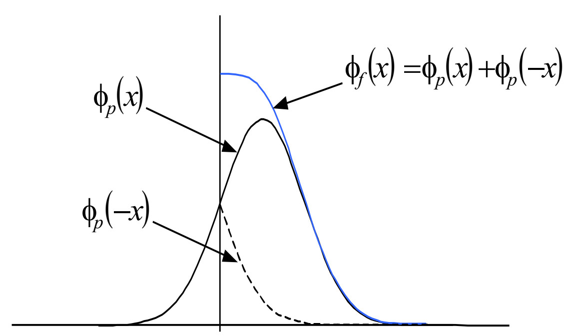


Figure 11

For example, if the minimum displacement has a p.d.f. $\phi_p(x)$ (the subscript p denotes the "parent" distribution) the minimum distance has a p.d.f. given by

$$\phi_f(x) = \phi_p(x) + \phi_p(-x) \quad \text{for } x \geq 0 \qquad (31)$$

where the subscript f denotes the "folded" distribution.

Also, if the parent distribution has a cumulative probability function[1] $\Phi_p(x)$ then the folded distribution has a cumulative probability function given by

$$\Phi_f(x) = \Phi_p(x) - \Phi_p(-x) \quad \text{for } x \geq 0 \qquad (32)$$

The cumulative probability function $\Phi(x)$ of the minimum distance, which is the area under the p.d.f. over the interval $(0, x)$, is the probability that the minimum distance is less than or equal to x.

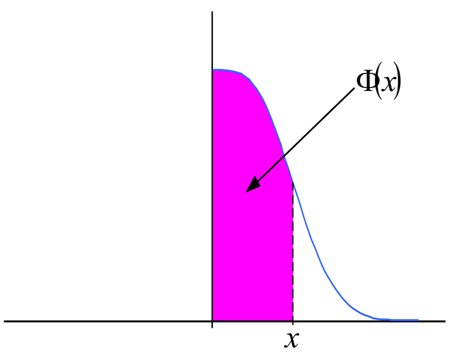


Figure 12

Under the assumptions which led to the minimum displacement having a normal distribution (equations (16) and (28)), the minimum distance has a "folded" normal distribution with probability density function given by

$$\phi(x) = \phi_{normal}(x) + \phi_{normal}(-x) \qquad (33)$$

where $\phi_{normal}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ is the probability density function of a normally distributed random variable.

The cumulative probability function for the minimum distance is given by

$$\Phi(x) = C\left(\frac{x-\mu}{\sigma}\right) - C\left(\frac{-x-\mu}{\sigma}\right) \qquad (34)$$

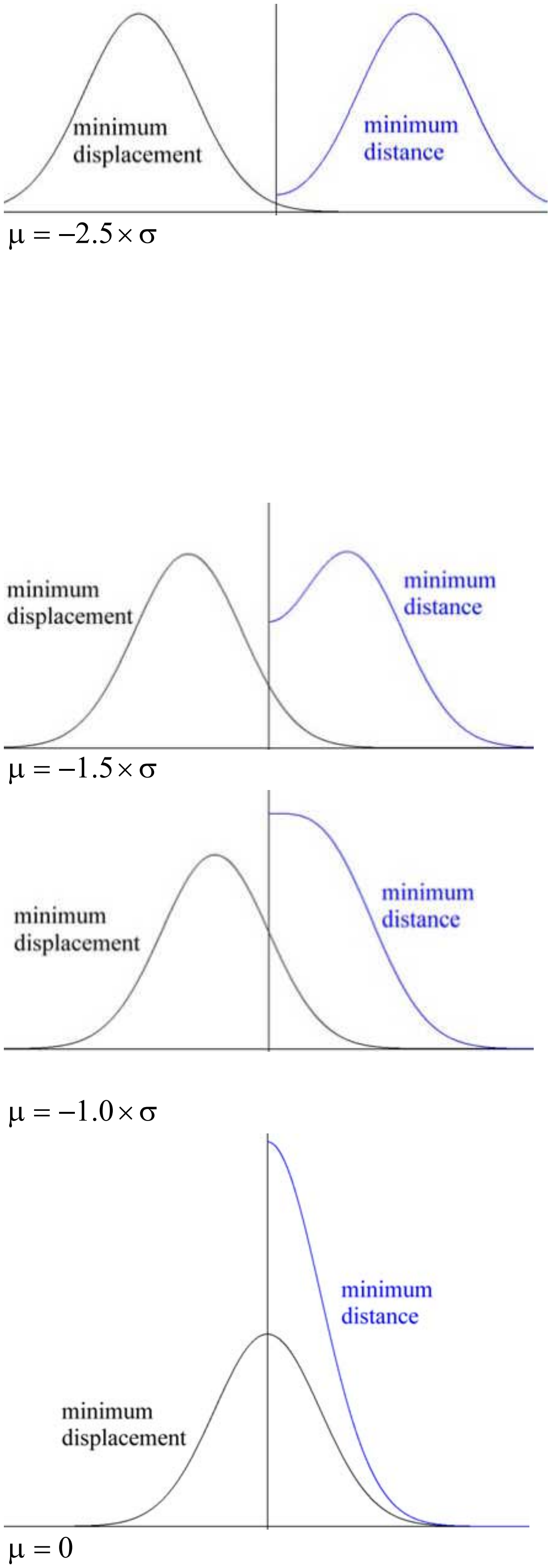
where $C(x)$ is the cumulative probability function of a normally distributed random variable with zero mean and unit variance (see equation (15)).

The probability of conflict (equations (17) and (29)) is the cumulative probability function of the minimum distance evaluated at the required separation, i.e., the probability that the minimum distance is less than the required separation.

The following sequence of images (figure 13) illustrates possible shapes of folded normal distributions in relation to their parent normal distributions, for various values of the mean of the parent distribution.

In the first and last cases, where the mean of the normal distribution is more than two standard deviations away from the origin, it is apparent that the distribution of the minimum distance is effectively normal.

[1] The cumulative probability function $\Phi(x)$ of a random variable X is the probability that X takes a value less than or equal to x, i.e., $\Phi(x) = P(X \leq x)$. For a continuous random variable the cumulative probability function $\Phi(x)$ is the area beneath the probability density function $\phi(x)$ over the interval $(-\infty, x)$.



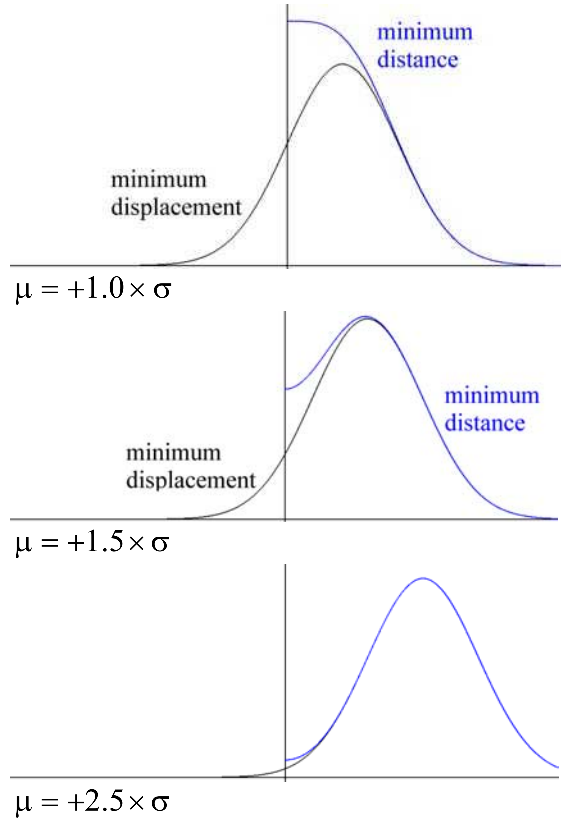


Figure 13

Finally, figures 14 and 15 illustrate minimum distance probability density functions, calculated using (33) with the mean and variance of the minimum displacement distribution as given in (28). The first set of graphs (figure 14) is for 90 degree crossing encounters, in which the predicted minimum distance is 3 nautical miles, i.e., a predicted conflict. Probability density functions are shown for a range of predicted times to minimum distance.

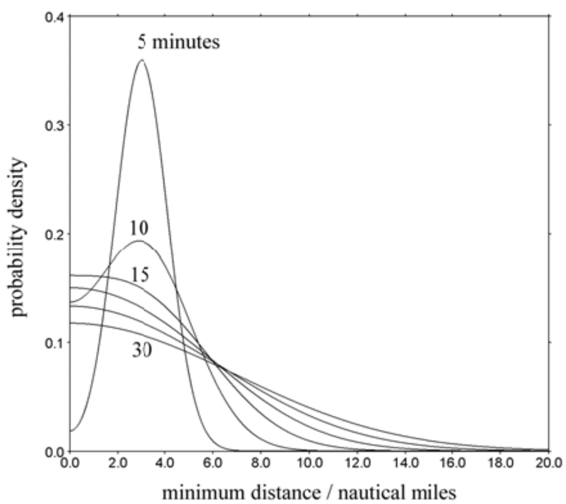


Figure 14 Minimum distance probability density functions (predicted minimum distance = 3 nautical miles)

As the time to minimum distance increases the probability density function flattens out. The area beneath the curve in the range (0, 5) nautical miles, that is, the probability of conflict, decreases and the area beneath the curve for minimum distances greater than 5 nautical miles, which corresponds to the false alert rate for encounters with this geometry, increases.

In the second set of graphs (figure 15) the predicted minimum distance is 7 nautical miles, i.e., a conflict is not predicted. Here again, as the time to minimum distance increases the probability density function flattens out. The area beneath the curve in the range (0, 5) nautical miles, that is, the probability of conflict, which here corresponds to the miss rate for encounters with this geometry, increases (at least up to about 25 minutes).

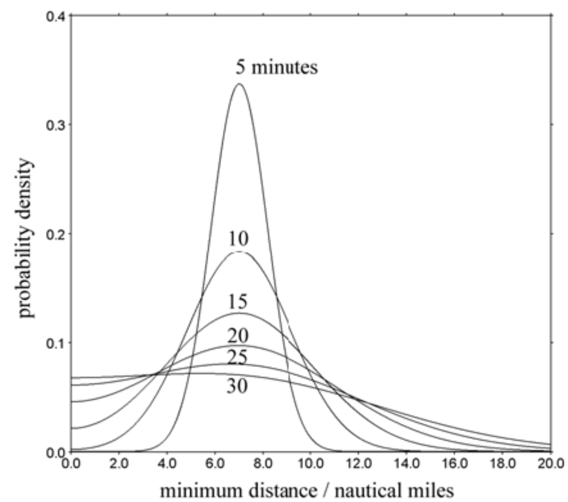


Figure 15 Minimum distance probability density functions (predicted minimum distance = 7 nautical miles)

6. Conclusion

A method of estimating conflict probability has been described. Initially the method was presented considering only along-track errors but subsequently cross-track errors were included. Graphs of conflict probability were produced which illustrate limits on the levels of performance that may be attained in medium-term conflict detection and resolution (in the absence of measures to reduce along-track errors). It was shown that, under certain assumptions, the minimum displacement of one aircraft from the other has a normal distribution, and that the minimum distance between them has a folded normal distribution.

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