Sequential Decision Model for the Single Airport Ground Holding Problem

Pei-Chen Barry Liu
Mark Hansen

Booz Allen Hamilton
University of California, Berkeley
Outline

- Introduction and background
- Model and formulation
- Computational strategy
- Concluding remarks
Ground Holding Problem

- Ground Delay Program
  - Mitigate destination airport capacity-demand imbalances by assigning delays to flights at their origin airports

- Ground Holding Problem (GHP)
  - Rationale: airborne delay is more expensive than ground delay
  - Approach: assign ground delays to flights
  - Goal: minimize Expected Total Delay Cost
    - minimize \{\text{ground delay cost} + E[\text{airborne delay cost}]\}

- Single Airport Ground Holding Problem (SAGHP)
  - SAGHP solves the GHP for one destination airport
Evolution of Models for the SAGHP

Response to Information Update

Multi-Stage Recourse
- Multi-Stage Recourse (up to scheduled departure time)
- Two-Stage Recourse

Static

Incorporation of Stochastic Factors
- Deterministic
- Scenario
- Scenario Tree
- Markov Process

Models:
- Terrab & Odoni '93
- Ball et al. '03
- Terrab & Paulose '92
- Richetta & Odoni '94
- Mukherjee & Hansen '05
Capacity Scenarios and Scenario Tree

Use of capacity scenarios and scenario trees in optimization models
- Static Model—capacity constraint
- Dynamic Model—capacity constraint and information update
Sample Capacity Scenarios at SFO

SFO2003 6 clusters (k = 6)

Cluster 1 (10%)
Cluster 2 (38%)
Cluster 3 (18%)
Cluster 4 (13%)
Cluster 5 (10%)
Cluster 6 (11%)

Time

AAR

Shortcomings of Scenario-based Methods

■ Fundamental problems
  □ Impose a finite-scenario tree structure on a reality where there is a much larger set of possibilities for capacity evolution
  □ Not to utilize improved information about future capacity which can be obtained continually rather than at a few discrete branching points

■ Issues found in empirical studies
  □ Costs incurred from applying the output of scenario-based optimization models is considerably higher than the theoretical optimization results
    ■ The actual capacities vary around the nominal values assumed in the optimization
    ■ Uncertainty in correctly identifying the scenario that matches best with the condition
Research Goal

- Improve the ability of air traffic managers to handle uncertainty and incorporate probabilistic forecast information in ground delay programs

- Learn from the shortcomings of the scenario-based models for SAGHP and explore scenario-free alternatives
  - Propose a scenario-free sequential decision model for the SAGHP
  - Develop computational strategies and demonstrate computational feasibility
Model and Formulation

- Markovian capacity evolution process
- Sequential decision model
- Dynamic programming formulation
- Algorithmic complexity
Suppose capacity evolution is Markovian with transition matrix:

\[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

Flight time: 1 period

Capacity at \( t = 2 \) is 1
Sequential Decision Model for the SAGHP

- Decision epoch
  - every quarter hour

- State
  - destination airport arrival capacity

- Available actions
  - number of flights to hold

- State and action dependent cost
  - ground and airborne delay cost

- State dependent transition probabilities
  - arrival capacity transition probabilities
Dynamic Programming Formulation

- **Optimality Equation**

\[
\begin{align*}
    f_t(K_t) &= \min_{H_t \leq G_t + D_t} \left\{ c_g H_t + c_a W_t + E[f_{t+1}(K_{t+1})] \right\} \\
    &= \min_{H_t \leq G_t + D_t} \left\{ c_g H_t + c_a W_t + \sum_{K_{t+1} = K_{\min}^{(t+1)}}^{K_{\max}^{(t+1)}} P_{K_t, K_{t+1}} \cdot f_{t+1}(K_{t+1}) \right\}
\end{align*}
\]

where \( G_{r+1} = H_t, G_0 = D_0, G_f = 0 \)

\( W_t = (L_t - K_t)^+ \)

\( c_g = \) cost of ground delay for one time period per flight.
\( c_a = \) cost of airborne delay for one time period per flight.
\( G_t = \) number of flights queued on ground by time \( t \).
\( D_t = \) number of flights scheduled for departure at time \( t \).
\( H_t = \) number of flights to hold on ground at time \( t \).
\( L_t = \) number of flights ready to land at time \( t \).
\( W_t = \) number of flights experiencing airborne delay at time \( t \).
\( K_t = \) arrival capacity at the airport at time \( t \).
\( K_{\max}(t) = \) maximum arrival capacity at the airport at time \( t \).
\( K_{\min}(t) = \) maximum arrival capacity at the airport at time \( t \).
\( P_{kk'} = \) transition probability from arrival capacity \( k \) to \( k' \) in the next period.
Formulation—flight specific holding

- Optimality equation

\[ f_t(K_t) = \min_{X_f^t, f \in F} \left\{ c_g \sum_{f \in F} (X_f^t - S_f^t) + c_d W_t + \sum_{K_{t+1} = K_{\max} (t+1)}^{K_{\min} (t+1)} P_{K_t, K_{t+1}} \cdot f_{t+1}(K_{t+1}) \right\} \]

s.t.

\[ Y_f^{t+1} = X_f^t - X_f^{t+1} \quad \forall f \in F, t = 0, \ldots, T - 1 \]

\[ A_t = \sum_{f \in F} Y_f^t \quad t = 0, \ldots, T \]

\[ W_t = (L_t - K_t)^+ \quad t = 1, \ldots, T; \quad W_0 = 0 \]

\[ L_t = A_t + W_{t-1} \quad t = 1, \ldots, T \]

\[ X_f^t \geq S_f^t \quad \forall f \in F, t = 0, \ldots, T \]

\[ X_f^t \geq X_f^{t+1} \quad \forall f \in F, t = 0, \ldots, T - 1 \]

\[ X_f^t \in \{0, 1\}, \quad Y_f^t \in \{0, 1\} \quad \forall f \in F, t = 0, \ldots, T \]

where

\[ X_f^t = \begin{cases} 1 & \text{if flight } f \text{ stays on the ground during time period } t \\ 0 & \text{otherwise} \end{cases} \quad \forall f, \forall t. \]

\[ Y_f^t = \begin{cases} 1 & \text{if flight } f \text{ is planned by the model to arrive in time period } t \\ 0 & \text{otherwise} \end{cases} \quad \forall f, \forall t. \]

\[ S_f^t = \begin{cases} 0 & \text{if flight } f \text{ is scheduled to depart in or before time period } t \\ 1 & \text{otherwise} \end{cases} \quad \forall f, \forall t. \]

\[ T = \text{the duration of flight of flight } f. \]

\[ A_t = \text{the number of flights planned by the model to arrive in time period } t. \]
Formulation—duration group-based holding

Optimality equation

\[ f_t(K_t) = \min_{0 \leq Z'_\gamma \leq G'_\gamma + S'_\gamma} \left\{ c_g \sum_{\gamma \in \Gamma} Z'_\gamma + c_d W_t + \sum_{K_{t+1} = K_{\min(t+1)}}^{K_{\max(t+1)}} P_{K_t, K_{t+1}} \cdot f_{t+1}(K_{t+1}) \right\} \]

s.t. \[ G'_\gamma = Z'^{-1}_\gamma \quad \forall \gamma \in \Gamma, \ t = 1, \ldots, T; \quad G'_0 = 0, \forall \gamma \in \Gamma \]

\[ \xi^{t+1}_\gamma = G'_\gamma + S'_\gamma - Z'_\gamma \quad \forall \gamma \in \Gamma, \ t = 0, \ldots, T - 1 \]

\[ A_t = \sum_{\gamma \in \Gamma} \xi^t_\gamma \quad t = 1, \ldots, T \]

\[ W_t = (L_t - K_t)^+ \quad t = 1, \ldots, T; \quad W_0 = 0 \]

\[ L_t = A_t + W_{t-1} \quad t = 1, \ldots, T \]

\[ Z'_\gamma \in Z^+ \quad \forall \gamma \in \Gamma, \ t = 0, \ldots, T - 1 \]

where \( \Gamma \) is the set of groups that represent the classification of flights.

\( Z'_\gamma \) = number of flights of group \( \gamma \) to hold on the ground in time period \( t \).

\( \xi^t_\gamma \) = number of group \( \gamma \) flights to arrive in time period \( t \).

\( S'_\gamma \) = number of group \( \gamma \) flights scheduled for departure at time \( t \).

\( G'_\gamma \) = number of group \( \gamma \) flights queued on ground from previous periods by time \( t \).
A Simple Example

Example:

# of Decision Stages: 2
Capacity Levels: 0 and 1
Transition matrix: \[
\begin{bmatrix}
0.6 & 0.4 \\
0.4 & 0.6
\end{bmatrix}
\]
Decision Node: eg. K0 = 0
Action Node: eg. H0 = 0
Terminal Node: eg. K2 = 0
# in queue on ground: Gt
# in queue in the air: Wt
Cg = 1, Ca = 3
Algorithmic Complexity

- Value iteration algorithm
  - Policy improvement (decision node)
  - Policy evaluation (action node)

- Complexity for duration group-based holding is $O\left(\left(\frac{F}{G}\right)^g N^T\right)$
  - $F =$ the number of flights to release in a decision stage
  - $G =$ the number of groups of flights
  - $T =$ the number of decision epochs in the planning horizon

- If there exists a priority ordering among the flights such that the grouping is not needed, the complexity will be $O((FN)^T)$
Computational Strategies

- Memoization
- Priority ordering
- Limited Search
Memoization—identify the overlapping subproblems

- Quintuple \((t, K_t, B_t, V_t, W_t)\) is necessary and sufficient to characterize a subproblem
  - \(K_t\): arrival capacity at \(t\)
  - \(B_t\): the bag of flight durations for the flights ready to be released in \(t\)
  - \(A_t\): the number of flights released and planned to arrived in \(t\)
  - \(V_t\): the vector of \(A_t's\) after \(t\) \(<A_{t+1}, A_{t+2}, \ldots, A_T>\)
  - \(W_t\): the length of the airborne queue at \(t\)
Priority Ordering

- Reduce the time complexity from $O((\frac{F}{G}^G N)^T)$ to $O((FN)^T)$

- Priority ordering schemes:
  - Longest Goes First (LGF): flight with longer duration has the priority
  - Ration by Schedule (RBS): flight with earlier scheduled arrival time has the priority
The cost-to-go function is convex in the number of flights to hold

\[
f_t(K_t) = \min_{0 \leq z^t_\gamma \leq K_t^{-1}, \forall \gamma \in \Gamma} \left\{ c_g \sum_{\gamma \in \Gamma} z^t_\gamma + c_a W_t + \sum_{K_{t+1} = K_{t+1}} P_{K_t, K_{t+1}} \cdot f_{t+1}(K_{t+1}) \right\}
\]

is convex in \( z^t_\gamma, \forall \gamma \in \Gamma \), where \( \Gamma \) is the set of duration-based groups of flights

- Non-negative integer decision variables (# of flights to hold)
- Piece-wise linear function
- Discrete & non-differentiable
  → Combinatorial optimization

When there is only one group?
Limited Search—cost-to-go with priority ordering

- Example: construction of cost-to-go function
  \[ f(x) = c_g x + p_1 c_a (2 - x)^+ + p_2 c_a (4 - x)^+ \]
  \[ y^+ \equiv \max(0, y) \]

- Example: cost-to-go function when priority ordering is adopted
  \[ f(x) = f(x_1 + x_2) = f_1(x_1) + f_2(x_2) \]
  \[ f_1(x_1) = c_g x_1 + 0.4 \cdot c_a (2 - x_1)^+ + 0.6 \cdot c_a (4 - x_1)^+ \]
  \[ f_2(x_2) = c_g x_2 + 0.7 \cdot c_a (1 - x_2)^+ + 0.3 \cdot c_a (3 - x_2)^+ \]

- Limited search heuristics
  - H1: Assume convexity, search from both ends
  - H2: Assume convexity, search from the lower end
Computational Result: Effect of Memoization

- Without memoization
- With memoization

Number of capacity levels: 3
Number of time periods: 6
Flight duration: 2 periods for each flight
Machine: Linux server

<table>
<thead>
<tr>
<th>Total Number of Flights</th>
<th>Computation Time without Memoization (milliseconds)</th>
<th>Computation Time with Memoization (milliseconds)</th>
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</tr>
<tr>
<td>9</td>
<td>60</td>
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</table>
Computational Result: Effect of Priority Ordering

Number of capacity levels: 3
Number of time periods: 6
Machine: Linux server

**Test Cases**

- Case 1: D=4: 100%
- Case 2: D=2: 50%, D=4: 50%
- Case 3: D=1: 25%, D=2: 25%, D=4: 50%
- Case 4: D=1: 25%, D=2: 25%, D=3: 50%, D=4: 25%
- Case 5: D=1: 25%, D=2: 25%
- Case 6: D=4: 25%

**Optimal Total Delay Cost**

- Case 1: D=4: 100%
- Case 2: D=2: 50%, D=4: 50%
- Case 3: D=1: 25%, D=2: 25%, D=4: 50%
- Case 4: D=1: 25%, D=2: 25%, D=3: 50%, D=4: 25%
- Case 5: D=1: 25%, D=2: 25%
- Case 6: D=4: 25%

**Computation Time (milliseconds)**

- Case 1: D=4: 100%
- Case 2: D=2: 50%, D=4: 50%
- Case 3: D=1: 25%, D=2: 25%, D=4: 50%
- Case 4: D=1: 25%, D=2: 25%, D=3: 50%, D=4: 25%
- Case 5: D=1: 25%, D=2: 25%
- Case 6: D=4: 25%
Computational Result: Effect of Limited Search

Number of capacity levels: 3
Number of time periods: 6
Machine: Linux server
Scenario-free Model with Real World Data

- Date: March 2\textsuperscript{nd}, 2006
- Location: SFO
- Transition matrix: 3 x 3
- Planning horizon: 7am to 2pm
- Flights affected: 116 flights departing between 7am and 12 noon
- Result:

<table>
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<tr>
<th>Priority Ordering</th>
<th>LGF</th>
<th>RBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Decision Nodes</td>
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<td>415228</td>
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<tr>
<td>Optimal Expected Total Delay Cost</td>
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<td>25.87</td>
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<td>Computation Time (milliseconds)</td>
<td>2324</td>
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The performance of a scenario-based model is compromised by a few shortcomings.

Development of scenario-free sequential decision model for SAGHP
- Dynamic programming formulation
- Computational strategies
  - Overlapping subproblems
    -> Memoization
  - Complexity reduction
    -> Priority ordering
  - Structural property
    -> Limited search

Computationally feasible for problems of real-world scale
Latest Development

- Comparison of scenario-based and scenario-free model
  - Theoretical comparison: equivalence and tradeoffs
  - Scenario-free model led to lower average incurred delay cost and lower variation in costs
  - Scenario-free model led to closer expected and incurred cost
  - Scenario-free model yields solutions that contain more balanced distribution of ground and airborne delay

- Future Work
  - Methodology for transition matrix estimation
  - Estimator of the computational cost
  - Other strategies to manage the complexity
  - Generalizability of the performance results
  - Accommodate different risk preferences using the scenario-free approach
  - Incorporation of the scenario-free approach into CDM
Lower Average Incurred Delay Cost
Lower Variation in Costs
Closer Expected and Incurred Delay Cost

Flight schedule:
03/21/2006 SFO, 8am to 12pm
Capacity profiles:
2003-2005, 1096 days
Delay Distribution: Scenario-Based Model
Delay Distribution
Scenario-Free Model
Realism of the Markovian Model (I)

![Graph showing the comparison between Simulation Generated and Field Data for Average AAR and SD(AAR) over time periods.](image)
Realism of the Markovian Model (II)

Lag = 4

Lag = 3

Lag = 2

Lag = 1

Correlation Coefficient

Simulation Generated

Field Data

Time Period

Field Data

Simulation Generated

Time Period

Correlation Coefficient

Time Period

Correlation Coefficient

Time Period

Correlation Coefficient

Time Period

Correlation Coefficient
Description of Test Cases

- Flight schedule for the test cases

<table>
<thead>
<tr>
<th>Flight Index</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
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- Transition matrix

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<td>0.4</td>
<td>0.4</td>
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</table>
Equivalence of Scenario-based Model and Scenario-free Model

- The models give the same result when the scenario-based model takes all possible scenarios from the Markov process as input.

- Verified numerically.

- For system with 3x3 transition matrices, 16 time epochs translate to $3^{16} = 43,046,721$ scenarios.

- Model selection
  - Scenario-based model is limited in the number of scenarios it can take.
  - Scenario-free model can solve problem with the above size in 2 minutes but it is computationally challenged by the combined force of factors in the time complexity.
Equity Considerations

- Ration by Schedule is considered equitable in air traffic flow management community
  - First In First Out

- Combining the previous results suggests the following implementation approach
  - Use RBS and LGF orderings
  - If RBS’s solution is as good (or almost as good) as LGF’s solution, then use it.
  - Else, if the gain from using LGF is big enough, use LGF’s solution.

- Weighted-score priority ordering based on flight’s schedule and duration