Throughput, Risk, and Economic Optimality of Runway Landing Operations

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Runway Data

Simultaneous Runway Occupancy

Landing Time Interval (sec)

What is the safe capacity of this runway?

Data Source

- DTW (Detroit) multilateration data
- 1 Week: 02-Feb-2003 thru 08-Feb-2003
- All runways (mostly 21L, 22R)
- Data restricted to
  - Peak-time landings (> 7 arrivals per qtr-hr)
  - IMC conditions
  - Aircraft weight category pairs with 3 nm separation
    (e.g., Large-Large)

Results of this talk can be generalized to varied fleet mix and weather conditions

Capacity

Capacity: Maximum achievable throughput on average

- Separation requirement: $S$ time units
- Assume
  - No gaps in arrival process
  - Arrivals are separated by exactly $S$

\[ \text{Capacity} = \frac{1}{S} \]

- Example: $S = 90$ seconds, Capacity $= 40 / \text{hr}$
- Problem: Separation standard not always met
A Revised Definition

- Choose target separation $T$ so that probability of separation violation is less than some small value.
- Restrict observations to peak periods
- Capacity = $1 / \text{Expected Separation}$
- “Buffer-adjusted” capacity

Runway Incursion-Based Capacity

- Determine target separation so that \( P\{LTI < ROT\} < \alpha \)
- Shift LTI distribution to the left or right
- Example: for \( \alpha = 10^{-4} \), increase separation by 15 sec

Risk vs. Throughput

- Use different safety thresholds $\alpha$ to evaluate risk versus throughput

A Risk-Free Capacity Definition

*Assume* the system is **completely** safe

- Simultaneous runway occupancy (SRO) is eliminated by go-around
- Assume pilot *always* takes go-around to avoid SRO
  (perfect information & execution)
Trade-offs

- Lowering the target separation allows more aircraft to land
- However, this also increases the rate of go-arounds
- At some spacing $T$, a maximum throughput is achieved
- Capacity = $1 / \text{Expected Separation}$
Simultaneous Runway Occupancy

\[
P\{SRO\} = P\{LTI < ROT \& \text{Trailing aircraft lands}\}
\]

\[
= P\{\text{Trailing aircraft lands} \mid LTI < ROT\} \cdot P\{LTI < ROT\}
\]

Enforced go-around

\[
= \text{Zero}
\]

\[
P\{LTI < ROT\} = P\{\text{Go Around}\} = p
\]

\[
P\{SRO\} = \text{Zero}
\]

**Definitions:**
- **LTI:** Landing Time Interval
- **ROT:** Runway Occupancy Time
- **SRO:** Simultaneous Runway Occupancy
Landing and Go-around Process

\[ \lambda = (1-p) \cdot \omega \]

\[ \lambda = \text{rate of successful landings} / \text{h} \]

\[ \omega = \text{rate of attempts} / \text{h} \]

\[ p \cdot \omega = \text{rate of go-around} \]

\[ \lambda = \text{rate of new arrivals} \]

\[ \omega = \text{rate of go-around} \]

\[ \lambda = (1-p) \cdot \omega + p \cdot \omega \]

\[ \lambda = \text{rate of new arrivals} \]

Goal: Maximize \[ \lambda(\omega) = [1-p(\omega)] \cdot \omega \]
Assumptions

- Distribution of time-separation unchanged along approach
- LTI and ROT of a lead-follower pair are independent
- Shifting LTI distribution to left or right does not change its shape
- Go-around is executed with perfect information
Maximizing Throughput

Successful Landings per Hour ($\lambda$)

Landing Attempts per Hour ($\omega$)

$\lambda = 39.6$

Prob. of Go-Around ($p$)

$\lambda$ (successful landing / hour)

$\omega$ (attempt / hour)
SRO and Wake Constraints

Simultaneous Runway Occupancy

- Underlying model structure is same
- Different constraints yield different functions:
  \[ \text{Prob\{Go-around to avoid SRO\}} = \text{Prob}\{LTI < ROT\} \]

Wake Vortex Hazard

- \[ \text{Prob\{Go-around to avoid wake hazard\}} = \text{Prob}\{LTI < x_0\} \]

\[ \text{Prob\{Go-around\}} = p(\omega) \]
With Wake Vortex Constraint

Objective: Maximize $\lambda(\omega) = [1-p(\omega)] \cdot \omega$

Assume wake separation requirement $x_0 = 65$ sec
Economic Optimality

Definitions

$R$: dollar benefit of a successful landing for all beneficiaries

$C$: expected average cost of a go-around

Maximize $ES(\omega; R, C) = [1 - p(\omega)] \cdot \omega \cdot R - p(\omega) \cdot \omega \cdot C$

Illustration:

• For DTW distributions under IMC, 3 nmi sep.
• Without wake constraint
• $C$ held constant
Optimal Capacity

Landings Equivalents per Hour
\( \lambda \) (Assume \( R=1 \))

<table>
<thead>
<tr>
<th>( x_0 = 65s )</th>
<th>( C/R )</th>
<th>( \omega^* )</th>
<th>( \lambda^* )</th>
<th>( P^* % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>36.8</td>
<td>33.6</td>
<td>8.7</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>33.7</td>
<td>32.8</td>
<td>2.6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>32.7</td>
<td>32.3</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>32.1</td>
<td>31.9</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>31.8</td>
<td>31.6</td>
<td>0.7</td>
<td></td>
</tr>
</tbody>
</table>
Dependency on Cost / Revenue

![Graph showing dependency on cost/revenue]

- **Wake Constraint**
  - $= 65$ sec
  - $= 70$ sec
  - $= 75$ sec

- **Landing Capacity**

- **Optimal Throughput (Landing/h)**

- **C/R**
Acknowledgments

- Wayne Bryant, Ed Johnson, NASA
- This talk solely represents the opinions of the authors
Summary

- Optimization model to maximize (without risk)
  - Throughput
  - Economic benefit
- Definition of capacity that
  - Takes into account statistical variation of arrival process
  - Does not depend on defining a safety level (e.g., $P(SRO) < 10^{-5}$)
- Models are notional and demonstrate principles
- Models generalize to non-uniform fleet mix
- Potential applications
  - Show capacity resulting from new technology (e.g., smaller variance in LTI)
  - Relative benefits of addressing wake technology and constraints vs. runway occupancy constraint
Backup Slides
Time Separations

2 nm (968 out of 1185pts)

1 nm (971 out of 1189pts)

0 nm (979 out of 1189pts)
Detroit Airport
Multi-lateration Data Collection

$Y(x)$ from one week operations of plane modes $= 11360066$
Sample Collection Process

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Threshold</th>
<th>Leave Runway</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy</td>
<td>10:23:14</td>
<td>10:24:04</td>
</tr>
<tr>
<td>Large</td>
<td>10:26:16</td>
<td>10:27:12</td>
</tr>
</tbody>
</table>

Airplane \(i+1\) approaches the threshold, and Airplane \(i\) leaves the runway.
Arrival Rates in every quarter hour, Runway 21L
### Total Observations in peak periods

- 4313 landings, 2 Feb 03 – 8 Feb 03 on all twelve runways
- 1862 in periods with arrival rate per quarter hour >= 7 (peak periods)

<table>
<thead>
<tr>
<th>a/c Type</th>
<th>03L</th>
<th>03R</th>
<th>04L</th>
<th>04R</th>
<th>09L</th>
<th>09R</th>
<th>21L</th>
<th>21R</th>
<th>22L</th>
<th>22R</th>
<th>27L</th>
<th>27R</th>
<th>Total</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Available</td>
<td>-</td>
<td>1</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>26</td>
<td>1.4</td>
</tr>
<tr>
<td>Small</td>
<td>-</td>
<td>19</td>
<td>26</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>98</td>
<td>0</td>
<td>3</td>
<td>101</td>
<td>18</td>
<td>17</td>
<td>280</td>
<td>15.1</td>
</tr>
<tr>
<td>Large</td>
<td>-</td>
<td>96</td>
<td>158</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>445</td>
<td>1</td>
<td>18</td>
<td>483</td>
<td>107</td>
<td>111</td>
<td>1418</td>
<td>76.2</td>
</tr>
<tr>
<td>B757</td>
<td>-</td>
<td>8</td>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>39</td>
<td>0</td>
<td>0</td>
<td>51</td>
<td>5</td>
<td>11</td>
<td>129</td>
<td>6.9</td>
</tr>
<tr>
<td>Heavy</td>
<td>-</td>
<td>0</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>0.4</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>124</td>
<td>206</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>594</td>
<td>1</td>
<td>22</td>
<td>643</td>
<td>131</td>
<td>141</td>
<td>1862</td>
<td>100</td>
</tr>
</tbody>
</table>

![Runway Diagram](image-url)
Comparison with ASPM Rates

**Average Difference:**
0.24 arrivals / qtr-h

**Standard Deviation:**
1.70 arrivals / qtr-h

**Total difference:**
160 landings or 3.6%

ASPM: Aviation System Performance Metrics
# Lead-Follow Mixes

## Percentage (out of 1805 pairs)

<table>
<thead>
<tr>
<th>Follow \ Lead</th>
<th>Small</th>
<th>Large</th>
<th>B757</th>
<th>Heavy</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1.7</td>
<td>12.5</td>
<td>1.2</td>
<td>0.1</td>
<td>15.5</td>
</tr>
<tr>
<td>Large</td>
<td>12.8</td>
<td><strong>58.8</strong></td>
<td>5.4</td>
<td>0.3</td>
<td>77.3</td>
</tr>
<tr>
<td>B757</td>
<td>0.9</td>
<td>5.4</td>
<td>0.6</td>
<td>0.0</td>
<td>6.9</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.1</td>
<td>0.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Sum</td>
<td>15.5</td>
<td>77.1</td>
<td>7.1</td>
<td>0.3</td>
<td>100</td>
</tr>
</tbody>
</table>
## Separation Minima Standards

<table>
<thead>
<tr>
<th>Follow\ Lead</th>
<th>Small</th>
<th>Large</th>
<th>B757</th>
<th>Heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Large</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>B757</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Heavy</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

1) Ref: FAA 7110.65 Separation Rules For Arrivals and departures
Landing Time Interval (LTI)

- LTI over the runway threshold
- Instrument meteorological condition (IMC)
- 3 nm pairs
- 523 samples (during IMC peak periods)
- Fit: Gamma\((40;11,6)\): mean 106 sec, std. dev. 27 sec.
Inter-Arrival Distance (IAD)

- IAD to the runway threshold
- Instrument meteorological condition (IMC)
- 3 nm pairs
- 523 samples (during IMC peak periods)
- Fit: Gamma(1.5;0.35,6): mean 3.6 nm, std. dev. 0.86 nm.
Independence of LTI

One-lag correlation coefficient: 0.25
- Correlation coefficients for higher degrees of lags are smaller
  - With some compromise, we decide the samples are independent
  - In similar manner, we accept sample independence for IAD.
• 669 samples for all aircraft types, peak IMC periods
• Sample mean 49.1 s, standard deviation 8.1 s
• Beta(6.1,15.4) in the (25,110)s
• N(49, 8.1^2) is rejected in the 0.10 significance level
**ROT: IMC vs. VMC**

- *ROT* for runways 21L/03R and 22R/04L
- IMC (590 samples), VMC (895 samples)
- No significant difference between IMC and VMC observed
Price of Risk

Assume DTW peak period IMC distributions and the safe WV separation of 60 s.

<table>
<thead>
<tr>
<th>$\omega$ (attempt/h)</th>
<th>37.30</th>
<th>40.00</th>
<th>42.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ (landing/h)</td>
<td>34.60</td>
<td>33.80</td>
<td>31.90</td>
</tr>
<tr>
<td>$p$</td>
<td>7.4%</td>
<td>15%</td>
<td>25.6%</td>
</tr>
<tr>
<td>Risked (landing/h)</td>
<td>2.70</td>
<td>6.20</td>
<td>11.00</td>
</tr>
</tbody>
</table>

In $\$ terms: Multiply the risky landings by $R$

For example, $2.7R$ if 37.3 is pushed without go-around!
Wake Vortex Cost

- $39.6 - 33.5 = 6.1$ Landing/hour
- $6.1 \times 10\,\text{h/day} \times 365\,\text{day} \approx 22,000\,\text{Landing/h}$
  (for a moderately busy runway!)
- $22,000 \times 2\,\text{runway} \times 35\,\text{airport} \approx 1,560,000\,\text{landing/\text{year}}$
- $1,000 \times 1,560,000 \approx 1.6\,\text{b}$
Generalized Model

Max. \( ES(\omega; R, C) = [1 - p(\omega)] \cdot \omega \cdot R - p(\omega) \cdot \omega \cdot C \)

\[ = \lambda(\omega) \cdot R - [\omega \cdot p(\omega)] \cdot C \]

\[ = R \left[ \lambda(\omega) - \frac{C}{R} \cdot \omega \cdot p(\omega) \right] \]

Let:

\[ g\left(\omega, \frac{C}{R}\right) = \lambda(\omega) - \frac{C}{R} \cdot \omega \cdot p(\omega) \]

Then:

\[ \text{Maximize } ES(\omega, R, C) \equiv \text{Maximize } g\left(\omega, \frac{C}{R}\right) \]
Model II: properties of Optimal Solution

Properties:
1. For a given $C$ and $R$, $g(\omega; C/R)$ are uni-modal
2. $g(\omega; C/R)$ decreases as $C/R$ increases for any fixed $\omega$
3. 
   \[ g \left( \omega, \frac{C}{R} \right) = \lambda(\omega) \cdot \frac{C}{R} \cdot \omega \cdot p(\omega) \rightarrow \lambda(\omega) \text{ as } \frac{C}{R} \rightarrow 0 \]
4. $\omega^*(C/R) = \text{Argmax}\{g(\omega; C/R)\}$ is decreasing in $C/R$ for $28 < \omega < \text{Argmax}\{dp/d\omega\}$

With WV effect

Without WV effect
Comparison of optimal $ES$ and $g$
Throughput in WV threshold

\[
\begin{align*}
\text{Lambda} &= 50.10 - 0.26 X \\
\text{Omega} &= 54.80 - 0.29 X
\end{align*}
\]
Optimal solution in terms of WV threshold

<table>
<thead>
<tr>
<th>wv thr (nmi)</th>
<th>wv thr (s)</th>
<th>w*</th>
<th>L*</th>
<th>p*</th>
<th>p^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.67</td>
<td>50</td>
<td>40.00</td>
<td>36.92</td>
<td>0.077</td>
<td>0.0059</td>
</tr>
<tr>
<td>1.83</td>
<td>55</td>
<td>38.71</td>
<td>35.81</td>
<td>0.075</td>
<td>0.0056</td>
</tr>
<tr>
<td>2.00</td>
<td>60</td>
<td>37.31</td>
<td>34.56</td>
<td>0.074</td>
<td>0.0054</td>
</tr>
<tr>
<td>2.17</td>
<td>65</td>
<td>35.82</td>
<td>33.22</td>
<td>0.073</td>
<td>0.0053</td>
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<tr>
<td>2.33</td>
<td>70</td>
<td>34.29</td>
<td>31.84</td>
<td>0.071</td>
<td>0.0051</td>
</tr>
<tr>
<td>2.50</td>
<td>75</td>
<td>32.73</td>
<td>30.49</td>
<td>0.068</td>
<td>0.0047</td>
</tr>
<tr>
<td>2.67</td>
<td>80</td>
<td>31.30</td>
<td>29.20</td>
<td>0.067</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

Graphs showing the relationship between p* and wv threshold (s), and p* and w (attempt/h).
Cost of Wake Vortex (in # of landings)

WV exists!
The difference between the optimal $\lambda$ with WV threshold and without any WV threshold provides the answer.

Example: If the absolute safe WV threshold is 60s then

WV cost = 39.7 – 34.6 = 5.1 landing/h