ATC Taskload Inherent to the Geometry of Stochastic 4-D Trajectory Flows with Flight Technical Errors

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Context

- Improved flow management concepts:
  - NextGen: “End-to-end strategic flow management”, “Self-separating flow corridors”
  - SESAR: “Time-based spacing and separation”
  - ICAO: “Optimized 4D trajectories”

- Some level of autonomy
- Varying precision requirements
- All these concepts need controller oversight to avoid severe degradation
Multilane flows

- Various stages of flight: enroute, approach,…
- Different autonomy and precision levels
Research Orientation

- Workload to meter a 4D flow structure:
  - Taskload: physical interventions
    - Impact of flow geometry and characteristics
  - Mental load: information requirements (in/out)
    - To maintain “good” traffic?
    - To improve “bad” traffic?
Research Question

• Workload to meter a 4D flow structure:

  • **Taskload**: physical interventions
    » Impact of flow geometry and characteristics

  • Mental load: information requirements (in/out)
    » To maintain “good” traffic?
    » To improve “bad” traffic?
Overview

Formal models
• Aircraft
• Flows
Overview

Statistical data

Formal models
• Aircraft
• Flows

Calibrated models
• Aircraft
• Flows
Overview

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Flow geometry

Monte Carlo simulations
Overview

- Statistical data
- Flow geometry

Formal models
- Aircraft
- Flows

Calibrated models
- Aircraft
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Monte Carlo simulations

Numeric probabilistic taskload
Overview

- Statistical data
  - Formal models
    - Aircraft
    - Flows
  - Calibrated models
    - Aircraft
    - Flows
  - Flow geometry
- Flow geometry
- Monte Carlo simulations
- Numeric probabilistic taskload
- Formal probabilistic taskload
**Step 1: Model Calibration**

- **Formal models**
  - Aircraft
  - Flows

- **Statistical data**

- **Calibrated models**
  - Aircraft
  - Flows

- **Flow geometry**

- **Monte Carlo simulations**

- **Formal probabilistic taskload**

- **Numeric probabilistic taskload**

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Popescu, Clarke, Feigh, Feron – ATC Taskload [...] with Flight Technical Errors – ATM Seminar 2011
Step 1: Model calibration

- Using statistical data to calibrate a stochastic model of the aircraft?
ICAO standards

• Area Navigation (RNAV):
  • Navigate any course within a mesh of beacons

• Required Navigation Performance (RNP):
  • Required navigation accuracy in a sector/route
  • Added performance monitoring and alerting specification to RNAV
ICAO standards

- Performance Based Navigation (PBN) concept:
  - FTE less than RNP level: 95% probability
  - FTE over 2 x RNP level: $10^{-5}$ probability

- Statistical consequence for a gaussian:
  - $\sigma \leq 0.45 \times \text{RNP}$
Plane motion

• Oscillations around a prescribed 4-D trajectory:
  • 3-D spatial requirements (RNP)
  • Associated temporal requirement (along-track)

• Traditional model: Brownian motion
  • Hu, Prandini, et al.
  • OK for short time
  • Unrealistic for long time (unbounded deviations)
Formal model

- Increase order of the model
- Ornstein-Uhlenbeck process:
  » mean reverting Brownian motion

\[ dX_t = \kappa (\mu - X_t) \, dt + \sigma dW_t \]
- \( X_t \) state = stochastic deviation
- \( \mu \) drift = deterministic deviation rate
- \( \sigma \) volatility, \( W_t \) Wiener / Brownian = nav inaccuracy
- \( \kappa \) elasticity = cockpit resistance to major deviations
Calibration method:

- Maximum likelihood estimation

- Available experimental data is insufficient
  - Statistical model in the literature: Johnson unbound system (Levy et al., 2003)
  - Calibrated O-U stochastic process (3 parameters) to match 4 statistical moments
  - Johnson random number generator
Outcome

• Simulated plane Flight Technical Error (O-U process)
  • ability to follow the defined path or track
Step 2: Simulation

Formal probabilistic taskload

Statistical data

Flow geometry

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Flow geometry
Step 2: Simulation

- Feeding the calibrated model into a simulation?
Flow geometry

- **Multilane flows**
  - Different RNP standards (aircraft types, flight regions, destinations)
  - Single overseeing controller
  - May or may not be in geographic proximity
  - High density enroute or parallel final approaches
**Outcome**

- **Taskload probability: multilane flow, 2 hours**
  
  » Lane 1 RNP-0.1; Lane 2 RNP-0.12; 
  Lane 3 RNP-0.15; Lane 4 RNP-2

- Most precise lane (RNP-0.1) is the main source of taskload
- Second lane increases expectation
- Additional more tolerant lanes have little effect

[Diagram showing probability over interventions over 2 hours]
Flow geometry

- Crossing

\[ \lambda_1, \lambda_2 \text{ flow intensities (aircraft/hr)} \]

\[ e_1, e_2 \text{ flow “extents” (RNP precision)} \]

\[ \alpha \text{ flow crossing angle} \]

\[ x_1, x_2 \text{ crossing zone} \]
Outcome

- Taskload probability: crossing, 2 hours
  - Scheduling conflicts create most of the taskload
  - Avoid crossing flows; when imperative, right angle should be favored

![Graph showing flow crossing 4-D control taskload](image1)
![Graph showing flow crossing scheduling conflict taskload](image2)
Conclusion

• Current precision supports PBN concept (performance based navigation):
  » Less than 10 interventions per 2h shift enroute
  » Flow crossings may pose scheduling problems rather than precision correction

• Applications:
  » Safety assurance in mixed RNP standards:
    » Parallel multilane enroute
    » Final approach
Acknowledgments

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Maximum likelihood

- Stochastic calculus:
  \[ \{dX_t = \kappa(\mu - X_t)dt + \sigma dW_t\} \Leftrightarrow \]

  \[ \forall s, t : 0 \leq s \leq t, X_t \text{ conditional upon } X_s \text{ is distributed as} \]

  \[ X_t \sim X_s e^{-\kappa(t-s)} + \mu \left(1 - e^{-\kappa(t-s)}\right) + \sigma \left[\frac{1 - e^{-2\kappa(t-s)}}{2\kappa}\right]^{\frac{1}{2}} \mathcal{N}(0,1) \]

  i.e. consecutive samples are almost affine

  \[ X_{i+1} = aX_i + b + \epsilon \]
Maximum likelihood

\[ X_{i+1} = aX_i + b + \epsilon \]

means \( \{X_{i+1} - aX_i - b\} \sim \mathcal{N}(0, \sigma'^2) \)

so the conditional density of \( X_{i+1}|X_i \) is a Gaussian:

\[
f_{X_{i+1}|X_i}(x) = \frac{1}{\sqrt{2\pi\sigma'}} e^{-\frac{(x-aX_i-b)^2}{2\sigma'^2}}\]
Least squares

\[ \epsilon \sim \mathcal{N}(0, \sigma^2) \]

- Max likelihood and linear least squares will coincide
- Linear least squares is fast and efficient
Least squares

\[ dX_t = \kappa (\mu - X_t) dt + \sigma dW_t \quad | \quad X_{i+1} = aX_i + b + \epsilon \]

\[ \kappa = -\frac{\ln a}{\delta t} \]

\[ \mu = \frac{b}{1 - a} \]

\[ \sigma^2 = \sigma^2_\epsilon \left( \frac{-2 \ln a}{\delta t (1 - a^2)} \right) \]

DONE
Maximum likelihood

• Recall model: \( dX_t = \kappa(\mu - X_t)dt + \sigma dW_t \)

• Method maximizes probability of obtaining samples over the class of parametric models:
  let \( \theta \stackrel{\text{def}}{=} [\kappa, \mu, \sigma] \)

• The most likely estimator of the parameters is given by
  \[
  \theta_{ml} = \arg \max_{\theta} \mathbb{P}(X_1, \ldots, X_n | \theta)
  \]
Maximum likelihood

1. In terms of density

\[ \theta_{ml} = \arg\max_{\theta} \mathbb{P}(X_1, \ldots, X_n | \theta) = \arg\max_{\theta} f_X(X_1, \ldots, X_n | \theta) \]

2. Assuming sample independence

\[ \theta_{ml} = \arg\max_{\theta} \prod_{i=1}^{n} f_X(X_i | \theta) \]
Maximum likelihood

• It’s simpler to introduce the log-likelihood function (notice variable ↔ parameter)

\[ \mathcal{L}(\theta | X_1, \ldots, X_n) = \ln f_X(X_1, \ldots, X_n | \theta) = \sum_{i=1}^{n} \ln f(X_i | \theta) \]

\[ \theta_{ml} = \arg\max_{\theta} \mathcal{L}(\theta | X_1, \ldots, X_n) \]
Maximum likelihood

\[ \theta_{ml} = \arg \max_\theta \mathcal{L}(\theta|X_1, ..., X_n) \]

- For \( f_{[X_{i+1}|X_i]}(x) = \frac{1}{\sqrt{2\pi}\sigma'} e^{-\frac{(x-ax_x-b)^2}{2\sigma'^2}} \) with \( \delta t \) timestep

\[
\mathcal{L}(\theta|X_1, ..., X_n) = -\frac{1}{2} \ln 2\pi - n \ln \sigma' \\
- \frac{1}{2\sigma'^2} \sum_{i=1}^{n-1} [X_{i+1} - X_i e^{-\kappa \delta t} - \mu(1 - e^{-\kappa \delta t})]^2
\]
Maximum likelihood

- Necessary (& sufficient since concave) for max

\[
\nabla \mathcal{L}(\theta | X_1, \ldots, X_n) = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial \kappa} = 0; \quad \frac{\partial \mathcal{L}}{\partial \mu} = 0; \quad \frac{\partial \mathcal{L}}{\partial \sigma} = 0
\]
• With the least squares method

\[ dX_t = \kappa(\mu - X_t)dt + \sigma dW_t \quad | \quad X_{i+1} = aX_i + b + \epsilon \]

\[ \sigma^2 = \sigma^2_{\epsilon} \left( \frac{-2 \ln \alpha}{\delta t (1 - a^2)} \right) \]
• With the least squares method

\[ dX_t = \kappa(\mu - X_t)dt + \sigma dW_t \quad | \quad X_{i+1} = aX_i + b + \epsilon \]

\[ \sigma^2 = \sigma_\epsilon^2 \left( \frac{-2 \ln a}{\delta t (1 - a^2)} \right) \]

• In practice how do we find \( \sigma_\epsilon^2 \)?
• With the least squares method

\[ dX_t = \kappa (\mu - X_t) dt + \sigma dW_t \quad | \quad X_{i+1} = aX_i + b + \epsilon \]

\[ \sigma^2 = \sigma^2_\epsilon \left( \frac{-2 \ln a}{\delta t (1 - a^2)} \right) \]

• In practice how do we find \( \sigma^2_\epsilon \)?

• By MAXIMUM LIKELIHOOD